

ERIC Document Resume		See Instruction Sheet		DATE OF RESUME 7/4/65	DATE RECEIVED
A. PUBLICATION DATA					
1. ERIC NUMBER ED002253		2. ORIGINATOR CONTROL NO(S). N.A.		3. IDENTIFICATION NO. N.A.	
4. AUTHOR(S): Johnson, Ronald E., and others				DO NOT WRITE IN SPACE BELOW	
5. TITLE OF DOCUMENT ABSTRACTED: Understanding the Slide Rule				MICROFILM COPY AVAILABLE YES ☒ NO ☐	
				COPYRIGHT GRANTED YES ☒ NO ☐	
				COPYRIGHT RELEASE FILED YES ☒ NO ☐	
6. PUBLICATION TITLE: Independent Learning Booklet				TYPE OF REPORT	
7. EDITOR(S): Mann, William		8. PLACE OF ORIGIN Department of Education San Diego County 6481 Linda Vista San Diego 11, California		PROPOSAL ☐	
9. PUBLISHER OR SOURCE Department of Education, San Diego County, San Diego, California				IN-PROGRESS ☐	
10. VOL. NA		11. PAGES N.A.		12. MO./YR. 1961	
13. TOTAL PP. 72		14. INST/LOC CODE		15. ERIC SATELLITE CODE	
B. INDEXING DATA					
14. SUBJECT AREA			15. TYPE OF ENDEAVOR Study Guide		
16. INDEXING TERMS (6-12)					
arithmetic curriculum		home study		students' interests	
advanced students		elementary education		slide rule	
independent learning		experimental approaches		logarithms	
				exponents	
				independent study	
				mathematics	
17. ABSTRACT (MAX. 250 WORDS)					
A booklet designed for elementary school students to be used independently from and in addition to the regular classroom curriculum in mathematics is given. The fifth- or sixth-grade student is presented with a discussion of the applications of the slide rule and with a background review of necessary concepts. The concepts of the slide rule are examined. Number scales, exponents, logarithms, decimal point estimation, and procedures used to multiply and divide on the slide rule are presented in logical sequence. The teacher should identify potential users, orient them to the area to be explored, and stimulate and maintain their interest in sustained independent effort.					
18. SOURCE OF ABSTRACT					
C. FUNDING DATA					
19. SUPPORTING ORGANIZATION		20. DIVISION OR DEPT.		21. CONTRACT NO.	
		22. TOTAL COST OF PROJECT		PROJECT NO.	
		23. DURATION OR PROJECT		U.S. Department of Health, Education, and Welfare Office of Education	

INDEPENDENT LEARNING BOOKLET

UNDERSTANDING
THE SLIDE RULE

DEPARTMENT OF EDUCATION • SAN DIEGO COUNTY

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INDEPENDENT LEARNING BOOKLET

Understanding the Slide Rule

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Published by

**DEPARTMENT OF EDUCATION, SAN DIEGO COUNTY
6401 Linda Vista Road
San Diego 11, California**

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Printed in U. S. A.**

TO THE TEACHER

This Independent Learning booklet is designed to help you open new avenues of learning for students who show a special interest in mathematics and a readiness to study on their own. Such students will be able to work directly from printed materials, using abstract ideas to follow a line of mathematical reasoning. Occasionally, of course, even the ablest of them will need your help to get over some of the rough spots, but your main concern in connection with the booklet will be to identify students who can benefit from using it, to orient them to the area to be explored, and to stimulate and maintain their interest in sustained independent effort.

Since the topic of the booklet trends outward from the regular curriculum, the challenge to do this independent work should arise from students' own searching in the field or from new interests that you have succeeded in awakening. Students will profit most if they elect voluntarily to do the work rather than receiving it as an assignment or as a means of earning extra credit.

The material is so organized that students may progress at their own best pace. They should be encouraged

to move ahead in discovering for themselves meanings and approaches in mathematics beyond those usually taught in the classroom, but there should be no requirement to complete parts or all of the booklet, or to follow a set schedule.

While serious effort will be necessary, students should find the work enjoyable because of their interest in the subject. They should gain a real sense of satisfaction through working individually or with other students in independently achieving understanding, overcoming temporary difficulties, and evaluating their own progress. Your encouragement and occasional direct help will of course be needed.

This booklet obviously is not aimed at developing complete understanding of an area of mathematics. Its purpose is rather to provide an introduction, or an approach novel to the student, which will create new interests and the desire to go on from here to further study in other sources.

A brief teacher's guide has been prepared to assist you in using this booklet with your students. Copies of the guide are distributed to your district office by the Department of Education, San Diego County. Additional copies are always available.

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INTRODUCTION

You know that this is an age of computing devices and rapid calculation. Did you ever wish you could use such devices to speed up your own arithmetic work? You can — by learning to understand and use a slide rule, the most widely used tool of rapid computation.

This booklet will introduce you to some basic ideas about the slide rule and help you get started in using it. Once you understand why and how the slide rule works you have a clear path ahead to develop skill through practice. And you will enjoy this practice because you will see yourself gaining in power to solve difficult problems with ease.

You will be learning directly from this booklet, so follow the explanations carefully. Your teacher will help you if you really get “stuck,” but don’t ask for help until you have worked through a section several times. You will feel more pleased with your accomplishments if you have achieved them on your own.

A word of advice: This booklet is divided into several parts. Some of them, at first, may seem too easy to bother with. Don’t let them fool you! You will need to understand each one before going on to the next if you are to learn to use the slide rule so that it will really save you time and energy. Answer the questions and work the examples first without peeking ahead at the answers so that you can tell whether you really understand.

To start using this booklet you will need paper, pencil, and a ruler. Later you will need a paper sliding rule, a table of logarithms, and a standard slide rule. Your teacher will have these for you when you are ready. If a standard slide rule is not available at your school, you may want to buy one of your own. You should choose a rule which will be useful to you in the future, preferably one 10” long and made of plastic rather than wood. A slide rule such as this costs about two to four dollars.

Since other boys and girls will be using this booklet, please do not write in it. Good luck and good sliding!

Recalling Some Ideas

To get off to a good start, let's first review some vocabulary and ideas with which you are already acquainted. Having them clearly in mind is important to your success in learning to use the slide rule.

Digits and Numerals The symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 are called **digits**. We use these ten digits in writing all the **numerals** in the decimal place-value system of notation. Just as words stand for ideas, so numerals stand for number ideas. The numeral 340 stands for the number idea we call "three hundred forty." The digits of 340 (3, 4, and 0) could be used to write different numerals, such as 304, 430, 403, etc., each of which is a distinct number idea. When a digit stands alone, for example a 2, a 5, or an 8, we may speak of it as a digit or as a numeral.

The Decimal Place-Value System of Notation

hundreds	tens	ones	tenths	hundredths
5	5	5	5	5

Five hundred fifty-five and fifty-five hundredths

The ones' place is the center of the decimal place-value system of notation. As we move to the left, each place value is multiplied by ten. As we move to the right, each place value is divided by ten. This relationship is shown in the chart at the left.

Multiplying and Dividing by Ten and Powers of Ten

In the numeral 555.55 the center is 5 ones. The 5 at the left of ones' place stands for 5×10 , or 50; the 5 at its right stands for $5 \div 10$, or .5, and so on.

It is so easy to multiply by 10 and by its powers (100, 1000, etc.) that it really isn't necessary to show the work. What happens when you multiply a decimal fraction by 10? Study the examples on the following page and see if you can figure this out.

	a	b	c	d	e	f
Multiplicand	.5	.9	.12	.48	.376	.759
	$\times 10$	$\times 10$	$\times 10$	$\times 10$	$\times 10$	$\times 10$
Product	5.0	9.0	1.20	4.80	3.760	7.590

Look at each multiplicand and each product. Do any of the **digits** change? No, except for the zero, the digit in each product is the same as that in its multiplicand. A 5 is still a 5, a 9 is still a 9, etc. What does change then? Yes, the **value** that each digit represents changes. Although the digit 5 is in both the multiplicand and the product of Example a, it is in **tenths'** place in the multiplicand and in **ones'** place in the product.

Now compare each product to each multiplicand. Does each digit in a product stand for ten times as much as it did in the multiplicand? The zeros at the right of the decimal point in the products here do not change the values of the number; we say that they are **not significant**.

Study the examples below and see what happens.

a.
$$\begin{array}{r} .34 \\ \times 10 \\ \hline 3.40 \end{array}$$

b.
$$\begin{array}{r} .34 \\ \times 100 \\ \hline 34.00 \end{array}$$

c.
$$\begin{array}{r} .34 \\ \times 1000 \\ \hline 340.00 \end{array}$$

What would be an easier way to multiply by 100? by 1000?

Compare d to a, e to b, and f to c.

d.
$$\overset{\curvearrowright}{.34}$$

e.
$$\overset{\curvearrowright}{.34}$$

f.
$$\overset{\curvearrowright}{.340}$$

Does shifting the decimal points, as indicated by the arrows, accomplish the multiplications? How many places to the right is the point shifted to multiply by 10? by 100? by 1000? Notice that a zero is annexed at the right of the 4 in Example f in order to have enough places in the numeral.

To divide by 10 or its powers we merely shift the decimal point to the left: $250 \div 10 = 25.0$; $25.0 \div 10 = 2.5$; $2.5 \div 10 = .25$. Since we shift the decimal point one place to the left to divide by 10, how many places would we shift it to divide by 100? by 1000?

Sometimes "to keep the balance" or to **compensate** we need to multiply or divide the terms of an example by 10 or powers of 10.

**Relationship
Between
Decimal and
Common
Fractions**

Decimal fractions, since they are an extension of our decimal place-value system of notation, have as a denominator 10 or a power of 10. Many common fractions can be written as decimal fractions and vice versa; $\frac{5}{10}$ can be written as .5, or .3 can be written as $\frac{3}{10}$. We can also change some common fractions to decimal fractions, and still keep their values the same (constant). For example, to do this we must change the denominator 5 in $\frac{3}{5}$ to 10 or a power of 10 before it can be written as a decimal fraction. We think $\frac{3}{5} = \frac{?}{10}$. Since we have multiplied 5 by 2 to get the new denominator 10, to compensate so as not to change the value of the fraction we must also multiply the numerator 3 by 2. $3 \times 2 = 6$, so we now have $\frac{3}{5} = \frac{6}{10} = .6$. If we wanted to change $\frac{3}{5}$ to hundredths, we would multiply each term of the fraction by 20 (because $100 \div 5 = 20$, or $5 \times 20 = 100$). $\frac{3}{5} \times \frac{20}{20} = \frac{60}{100}$ or .60, and .6 and .60 have the same value. The decimal fraction .60 is read "sixty hundredths" or "point six zero"; .6 is read "six tenths" or "point six." You may want to refer to your textbook to review reading and writing decimal fractions.

**Ways to
Express
a Number**

A number can be expressed in many different forms, that is, we can take numbers apart and put them together in a variety of ways. For example, 8 can be expressed as $9 - 1$, $3 + 5$, $3\frac{1}{2} + 4\frac{1}{2}$, $16 \div 2$, $\frac{8}{1}$, 2×4 , $2 + 2 + 2 + 2$, and so on.

Or, a numeral such as 243 can be thought of as 2 hundreds, 4 tens, 3 ones; or as 24 tens, 3 ones; or as 243 ones. It can also be expressed as $200 + 40 + 3$, or as $(2 \times 100) + (4 \times 10) + (3 \times 1)$, and so on.

**Addition
and Multi-
plication**

In addition you combine quantities to find a total called the sum. Multiplication is the combining of equal quantities (addends) to find a total called the product. You can see the relationship of addition to multiplication in these examples: $2 + 2 + 2 = 6$, therefore 3 twos are 6; $3 \times 2 = 6$. Or, $3 + 3 = 6$; 2 threes are six; $2 \times 3 = 6$.

**Addition
and Sub-
traction**

Subtraction is the inverse, that is, the opposite or undoing, of addition. You can see the relationship of addition to subtraction in this illustration:

$2 + 3 = 5$ and $3 + 2 = 5$, therefore $5 - 3 = 2$ and $5 - 2 = 3$.

**Division
and Sub-
traction**

Division can be thought of as a process of finding how many times one number can be taken from another or how many times it is contained in another number. It is a shortened form of subtracting equal quantities from a total called the dividend.

For example, how many 2's can be taken from 8?

$$8 - 2 = 6 \quad 6 - 2 = 4 \quad 4 - 2 = 2 \quad 2 - 2 = 0$$

We can take 4 twos from 8; or we can think: There are 2 fours

in 8, or $8 \div 2 = 4$ or $2 \overline{)8}$. A division example can also be written as a common fraction: $\frac{8}{2}$

**Division
and Multi-
plication**

Division is the inverse of multiplication. You know that in **multiplication** you combine 4 threes to get the product 12. Since 4 threes equal 12, you can take 4 threes out of 12. We say that $4 \times 3 = 12$, therefore $12 \div 3 = 4$.

**Estimating
and
Approximating**

When we **estimate** in arithmetic we use our best judgment about an answer without working it out exactly; that is, we **approximate** the answer, or approach it closely. Estimating and approximating help us judge the reasonableness of the results of calculation. You will learn more about estimating in this booklet.

Rounding

Rounding numbers is helpful in estimating; when we "round" we approximate the value of the number. For example, if we want to estimate the quotient of $32.13 \div 5.1$ we can think of 30 divided by 5 and estimate that the quotient will be close to 6 (the exact quotient is 6.3). You may want to study rounding numbers in your textbook.

UNDERSTANDING THE SLIDE RULE

PART I

Number Scales and Sliding Rules

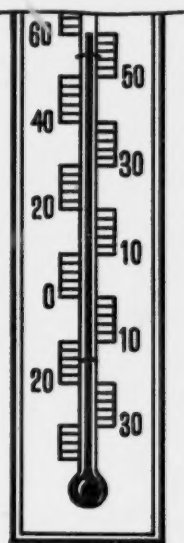
Have you ever heard of a number scale? Have you ever used one? If you have ever used a ruler or a thermometer you have used a number scale, although you may not be familiar with the use of the word **scale** in this connection.

How are a ruler and a thermometer alike? You say that both are measuring instruments and that numerals appear on both of them. That's correct. Do both have a system of marks? Study Figure A and Figure B. Are the marks on the ruler equally spaced? What about the marks on the thermometer?

Figure A



Figure B



The intervals between the marks on the ruler are equal, and the intervals between the marks on the thermometer are equal, aren't they? Do equal numerical values correspond to the equal intervals of each scale? That's right; on the ruler each equal distance has a numerical value of one. On the thermometer each equal distance has a numerical value of two. The ruler and the thermometer are examples of a uniform scale, a scale with equal intervals.

SET I

An example of scales that are used to represent number values on a line are shown in Figures C, D, and E. How are these number scales alike? How are they different? You may want to list all of the responses that you think of before you check your thinking on page 54.

Figure C

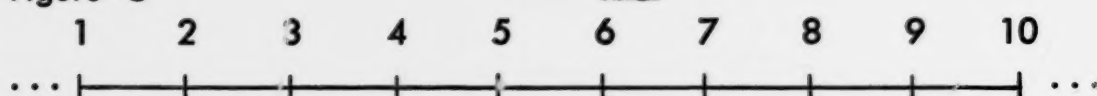


Figure D

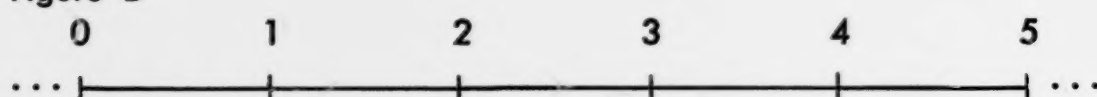
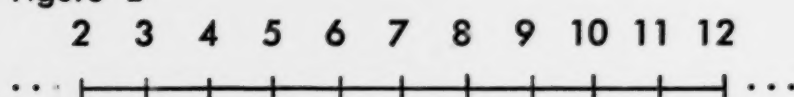
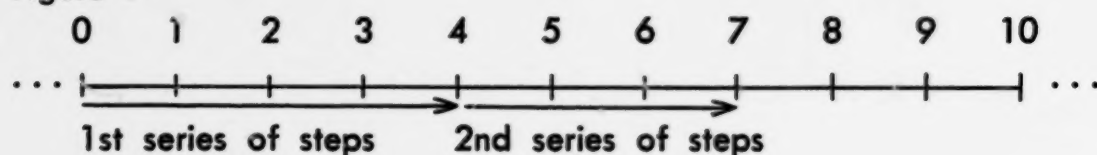


Figure E



A number scale can be used to add and subtract; in fact, even to multiply and divide. Let's find out more about this. We might think of adding on a number scale as similar to taking a series of steps of given distances (the addends) from one point to another and arriving at a stopping place (the sum). For example, Joe took 4 steps toward the front of the room. He then took 3 more steps in the same direction and stopped. How many steps did he take all together? Let's look at Figure F to see how this problem would be worked on a number scale.

Figure F



Do you see how the arrows picture the movement along the scale? What is the number name for the point where we arrive? Does $4 + 3 = 7$?

Make up some addition examples of your own and find their sums by using a number scale. You can draw your own lines.

Can you think of a way to subtract on the number scale? Since subtraction is the opposite or “undoing” of addition, we can subtract either by “building up” or by “taking away.” If we know the sum (the minuend) and one addend (the subtrahend) we can find the other addend (the remainder or difference). For example, Jill is riding her bicycle 5 miles to Joan’s house. She has already gone 3 miles. How many more miles does she still have to go? We can start at 5 and move back 3 (taking away) to arrive at 2, the difference, $5 - 3 = 2$; or, we can start from 3 and move ahead (build up) to 5 by counting—“four, five.” $3 + \text{what equals } 5$? $3 + 2 = 5$. Can you see that the difference between 5 and 3 is 2 in either case?

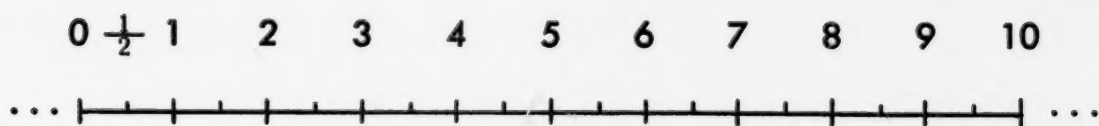
Draw lines and do some subtraction examples of your own. Then draw number scales to work the problems in SET II to be sure you understand how to add and subtract with a number scale. Check your thinking with the diagrams on page 55.

SET II

- Mary is 9 years old. Her brother is 13 years old. What is the difference between their ages? Do you have to start this scale with 0?
- Jack and Ray walked 4 miles in the morning and 7 miles in the afternoon. How many miles did they walk all together?
- How much longer is Eric’s rope than Bob’s? Eric has a rope 21 feet long, and Bob has a rope 18 feet long. Can you start this number scale with 13? with 18?

Have you already figured out how to add and subtract fractions on a number scale? Do we have to do anything more to the number scale? Add any more numerals? If so, what kind? Yes, we need to divide each space into equal parts and label each part with a fractional number name as we have started to do in Figure G below.

Figure G



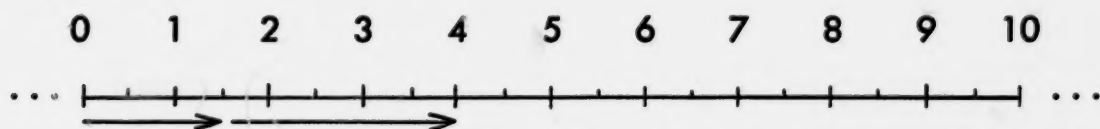
Do you notice that we have divided each space on Figure G into two equal parts, or into two halves? Start at the point labeled $\frac{1}{2}$ and count along the line to the right; begin, $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, etc. Could we have labeled each half as $\frac{5}{10}$ or .5? Why? What are some other number names for $\frac{1}{2}$?

Could we divide each half into two equal parts again? If we did this, how would we label the first dividing line at the left? Yes, $\frac{1}{4}$, or if we use decimal fractions, .25. Do you know why? Is $\frac{25}{100}$ or .25 the same value as $\frac{1}{4}$? You may want to use your textbook to review the relationship between decimal fractions and common fractions.

Could we divide each $\frac{1}{4}$ into half again? Would each of these spaces have a value of $\frac{1}{8}$? Why?

Are you ready to add fractions on the number scale? Study Figure H below to see how to add $1\frac{1}{2}$ and $2\frac{1}{2}$.

Figure H



Does $1\frac{1}{2} + 2\frac{1}{2} = 4$? Try adding fractions on a number scale using some examples of your own. Can you estimate a location on a number scale if it isn't labeled?

Do you remember how to subtract whole numbers on a number scale? From this, can you figure out for yourself how to subtract fractions on a number scale? Use a number scale to do this example, $5\frac{1}{2} - 3\frac{1}{4}$, both by "building up" and by "taking away."

Try a few examples of your own. Then do SET III to be sure you understand how to add and subtract fractions on a number scale.

SET III

Use number scales which you have drawn to do the following examples. Do Examples b and d both by "building up" and by "taking away."

a. $2\frac{3}{4} + 1\frac{1}{4} =$ b. $4 - 2\frac{1}{4} =$ c. $3\frac{1}{2} + 1\frac{1}{4} =$ d. $3\frac{1}{4} - 1\frac{1}{2} =$

Check your thinking with the diagrams on page 56.

Multiplying and Dividing with a Number Scale

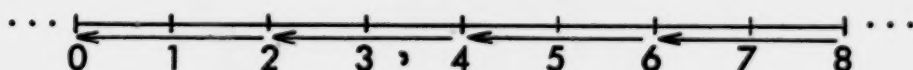
You have been adding and subtracting on a number scale. Can you figure out how to use a number scale to multiply and divide? (Actually, a real slide rule is used to multiply and divide rather than to add and subtract.) You know that we can think of multiplication as addition of equal addends. For example, 6×2 means the sum of six 2's. Can you visualize with arrows on the number scale the product of 3×4 ? ($4 + 4 + 4$).

Draw a number scale to illustrate 3×4 . Is your product 12? Does this help you see how multiplication and addition are related?

Multiplication on the number scale can be shown as steps of equal size to the right. The multiplicand (4) determines the size of the step and the multiplier (3) determines the number of steps. Now draw a number scale to show the products of some examples of your own.

Look at the number scale you drew to illustrate 3×4 . Can you also use it to find how many 4's can be taken out of 12; that is, to answer the problem $12 \div 4 = ?$ Moving along the scale from right to left, can you see that we can take three 4's from 12? Does this help you see that division can undo multiplication? Study Figure I to find out how many 2's can be taken from 8. $8 \div 2 = ?$

Figure I



$$8 \div 2 = 4 \quad \begin{array}{r} 4 \\ 2 \overline{)8} \end{array}$$

You can take four 2's out of 8. Division on the number scale is shown as steps toward the left. The dividend determines the starting point; the divisor determines the size of each step. The quotient is the number of steps.

Just as multiplication can be thought of as the addition of equal addends, division can be thought of as the subtraction of equal numbers. Use a number scale to subtract 3 from 9 three times ($9 - 3 - 3 - 3 = ?$). Then use a number scale to divide 9 by 3 ($9 \div 3$). Do you see how subtraction and division are related?

Use a number scale to do the examples in SET IV.

SET IV

a. $5 + 5 + 5 =$

b. $15 \div 5 =$

c. $3 \times 5 =$

d. $15 - 5 - 5 - 5 =$

Check your thinking with the diagrams on page 57.

Are you thinking, "This is all very interesting, but it's a slow way to add and subtract. I thought I was going to learn a fast way to do arithmetic." You are; it won't be long now. In fact, one of the ways that is quicker you are going to learn right now—how to use a **sliding rule**. (See illustration on page 57.)

Adding and Subtracting on a Sliding Rule

A quicker way to add and subtract is by using a paper sliding rule. Your teacher has one for you. When you get it, notice that it has two parts; one part is labeled A and the other part is labeled B. In discussing the use of this rule we will refer to these parts as the A scale and the B scale.

Let's use your sliding rule to do the simple example, $3 + 5$.

Step 1 Find the numeral 3 (first addend) on the B scale.

Step 2 Place the zero end of the A scale directly above the 3 on the B scale.

Step 3 Find the numeral 5 (other addend) on the A scale.

Step 4 Look at the numeral on the B scale directly below the 5 to find your sum. Is the numeral 8?

Is 8 the sum of 3 and 5? Try some more examples of addition on your own.

Even though this sliding rule has only 10 whole spaces, we can use it to add two-place numbers, but first we must learn more about reading the numerals on the rule.

Look at your sliding rule again. Notice that both the A and B scales are divided into ten large spaces. Each space is labeled with a numeral—1 through 10. We can consider that each of these ten spaces is a single whole; for example, the space between 1 and 2 is one whole, so is the space between 2 and 3, 4 and 5, etc. Each scale consists of 10 wholes.

Do you see that each of these ten whole spaces is divided into smaller spaces by lines of varying lengths? These are

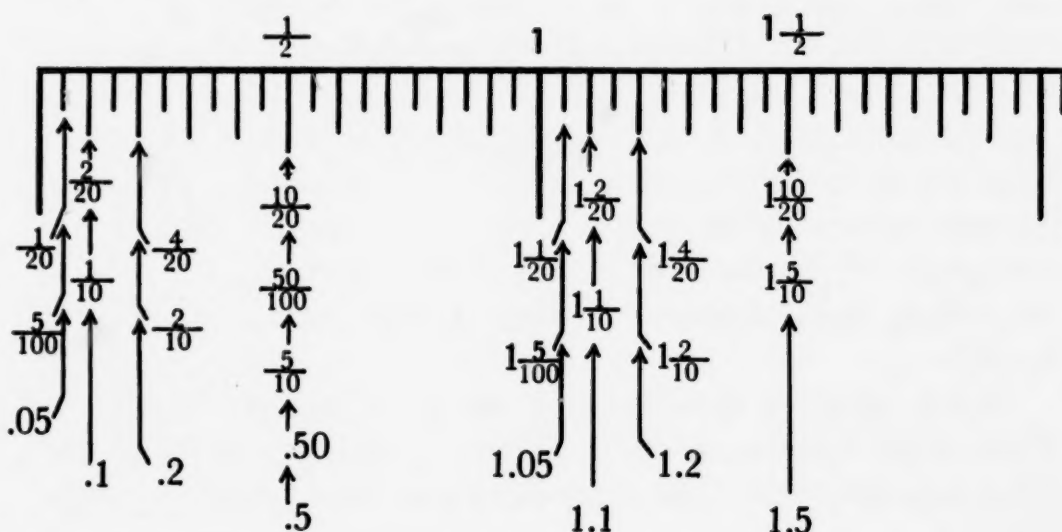
graduation marks to indicate the division of each of the 10 whole spaces into halves, tenths, and twentieths.

From 0 move one of the smallest spaces to the right. What part of the whole space does this line represent? Yes, you have moved $\frac{1}{20}$ of the whole space; this line (mark) could be labeled $\frac{1}{20}$. If we were going to label it with a **decimal** fraction, which number name would we give it? One twentieth can't be written as a decimal fraction in tenths, so it has to be written in hundredths. $\frac{1}{20} = \frac{5}{100}$. Remember our review on page 7? Now, how do you write five hundredths as a decimal fraction? How many places are there to the right of the decimal point? Do you need to annex a zero?

Moving along to the next line or mark to the right, we have moved another twentieth. We can call this line $\frac{2}{20}$ or $\frac{1}{10}$. Why? Does $\frac{1}{10} = .1$? One tenth equals how many hundredths? $\frac{10}{100} = .10$, doesn't it? Is .10 twice as much as .05? All together, how many twentieths are there in one whole space? How many tenths?

Figure J below illustrates part of the scale on your sliding rule. For illustration, some of the lines are labeled with their number names. Study Figure J carefully to be sure you can identify and label each line on your sliding rule. Could you label with number names the other lines on this scale?

Figure J (Enlarged view)



Before you go ahead, you should be able to locate the position on the scale of any number you might have in a problem. Notice that the lines indicating the wholes are longest. Next longest are those indicating the halves. Then come the lines indicating tenths, and the shortest are those indicating twentieths. Each of these lines that mark the divisions is called a graduation. By now you probably understand that all scales—foot rulers, thermometers, sliding rules—are divided into intervals or grades.

Practice reading in sequence (counting order) the graduations between the zero end and 1 on Figure J, using both decimal and common fractions. Begin:

$\frac{1}{20}$ or $\frac{5}{100}$ (.05); $\frac{2}{20}$ or $\frac{1}{10}$ (.1); $\frac{3}{20}$; $\frac{4}{20}$ or $\frac{2}{10}$ (.2); etc. Then read in sequence the graduations from 1 to 2. Begin:

$1\frac{1}{20}$ or 1.05; $1\frac{2}{20}$ or 1.1; etc.

We have seen that it is easy to add one-place numbers on our sliding rule, but what about two-place numbers, considering that we have only 10 whole spaces on each scale? Can you figure out how we could do it? Mathematicians have worked out a way. They call this “changing indexes.” With our original scale of 10 whole spaces, we have been using an index of 1, so $1 \times 1 = 1$, $1 \times 2 = 2$, etc. But if we were to use an index of 10, we would multiply each whole number on the scale by 10; thus, $10 \times 1 = 10$, so we would call the large 1 on our scale “ten.” The 2 we would call “twenty.” The 3 would become what? How about the 4? If we should use an index of 100, 1 would become 100. What would 2 become? or 7?

How would you locate a number such as 12 on our sliding rule using an index of 10? It would be farther to the right than 1 but not as far as 2, wouldn't it? You can ask, Ten times what number equals 12? $10 \times 1.2 = 12$, right? Or, 12 divided by 10 equals 1.2. (You may want to refer back to the section “Recalling Some Ideas” to review multiplying and dividing by 10.)

What about a number such as 12.5? You can think: 10 times what number equals 12.5, or 12.5 divided by 10 equals what number? Yes, 1.25. Can you locate this position on Figure J?

Using an index of 100, where would you locate a number such as 120? One hundred times what number equals 120, or $120 \div 100 = ?$ Yes, $120 \div 100 = 1.20$ or 1.2. Can you see that by changing to an index of 10, 1.2 can become 12, with an index of 100 it can become 120, or with an index of 1000 it can become 1200?

With an index of 100 where would you locate 125 on your sliding rule? $125 \div 100 = 1.25$. With different indexes 1.25 can be 12.5, 125, 1250, etc. Notice that the indexes used are all powers of 10. Can you see why? (You may want to refer to "Recalling Some Ideas" to review powers of ten.) But remember, whatever index you use, it must be the same for all the numbers in any one example. Why?

When you are sure that you understand the location of various numbers on the scale, use your sliding rule to locate the positions in SET V. Obviously we can't have enough lines on our rule to show every number so sometimes we have to estimate locations between the lines or marks.

SET V

a. 2	b. 20	c. 200
d. 1.5	e. 15	f. 150
g. 1.35	h. 135	i. 13.5
j. 23	k. 2.3	l. 230
m. 2.25	n. 225	o. 22.5

Check the index you used and the locations you found with the diagram on page 58.

• Now, back to adding two-place numbers. Try using your rule to add 14 and 65. What index will you have to use? Yes, 10. Since $14 \div 10 = 1.4$ and $65 \div 10 = 6.5$, can't you do this in your head simply by shifting the decimal point?

- Step 1** Locate the first addend, 1.4, on the B scale.
- Step 2** Place the zero end of the A scale directly over 1.4 on the B scale.
- Step 3** Locate 6.5, the other addend, on the A scale.
- Step 4** Below this 6.5 look at the mark on the B scale to find the sum. Can you read this on the scale even

though it is not labeled with the exact numeral? What number is it? Is it 7.9? Yes. But is 7.9 the answer to $14 + 65$? No. Since we **divided** both addends by 10 in order to locate them on our sliding rule, we must now make up for, or compensate for, this division by **multiplying** the sum that we read on the B scale by 10 to get our true answer. $7.9 \times 10 = ?$ Does $14 + 65$ equal 79?

Do this example again without referring to the above steps. If you can do it quickly and accurately, continue with SET VI B. If you feel you need more practice first do SET VI A and then SET VI B.

SET VI A

- | | |
|------------------|----------------|
| a. $2 + 3 =$ | b. $20 + 30 =$ |
| c. $200 + 300 =$ | d. $12 + 14 =$ |
| e. $120 + 140 =$ | f. $15 + 34 =$ |

Check your thinking on page 59.

SET VI B

Work the following examples on your sliding rule. After you do each example, check your work by adding with a pencil and paper. The answers in both cases should be the same.

- | | | |
|--------------------|--------------------|--------------------|
| a. $1.5 + 3.4 =$ | b. $45 + 22 =$ | c. $32 + 47 =$ |
| d. $6.1 + 1.7 =$ | e. $235 + 460 =$ | f. $1.04 + 7.35 =$ |
| g. $3.25 + 5.00 =$ | h. $6.75 + 3.20 =$ | i. $800 + 100 =$ |
| | j. $85 + 675 =$ | |

Did you notice that Example j is a bit different from any that you have had? Can you figure out how to do it? Try it and then read the explanation on page 60.

None of the examples which we have done so far have involved **carrying** into the next place; that is, in adding $14 + 65$ you do not have to carry any tens to the tens' column (place) from the ones' column, or any hundreds to the hundreds' column from the tens' column. But in an example such as $15 + 65$, you do carry. You add ($5 + 5 = 10$), then carry the 1 ten to the tens' column and add 1 (the carried number) $+ 6 + 1$.

Can you glance quickly at an example and tell whether or not there will be carrying? Take the example $15 + 18$. Looking at the numerals of both addends in the ones' column we can immediately see that $5 + 8$ will equal more than 9, so we know that we will have to carry.

Let's try the example $5 + 8$ (which involves carrying into the next decade, that is, from the ones' place to the tens' place). We will first do it as we have done the other examples to see what happens.

- Step 1** Place the left (zero) end of the A scale directly over the 5 (the first addend) on the B scale.
- Step 2** Notice that the 8 on the A scale is beyond the end of the B scale.

We can't add this way. Let's try another way.

- Step 1** Place the right, or 10 end of the A scale directly over the 5 (first addend) on the B scale.
- Step 2** Find the 8 on the A scale. Look directly below the 8 and find 3 on the B scale. (Obviously 3 isn't your answer.)
- Step 3** Notice that 5 spaces of the A scale extend over the left end of the B scale. When this 5 is added to the 5 (the first addend) on the B scale, the sum is 10. This 10 must be added to the 3 on the B scale to find the sum 13. Does $5 + 8 = 13$?

Do you understand why this works? You can do it without understanding it, but you are more efficient when you do understand. Remember in the section, "Recalling Some Ideas," we showed you some ways to break down or to express a number? Well, this is one way to illustrate what we just did with the sliding rule. First we broke down the addends, then we added them. For example, we actually broke the 8 down into $3 + 5$ so that this 5 could be combined with the other 5 to make 10. This gave us as addends $5 + 5 + 3$ instead of $5 + 8$. Does $5 + 5 + 3 = 5 + 8$? Does $5 + 5 + 3 = 10 + 3$?

Now try on your own a similar example, $9 + 7$, to be sure that you understand. If necessary, refer to the preceding three steps on this page.

Can you figure out a shorter way of doing this? Reviewing how we arrived at the answers to both of these examples with carrying, we find that we used the following steps:

- Step 1 We used the right (10) end of the A scale instead of the left (zero) end.
- Step 2 We placed the right end of the A scale over the first addend on the B scale.
- Step 3 We added to this addend whatever part of the A scale extended over the B scale to make 10.
- Step 4 We found the second addend on the A scale and looked directly below it on the B scale to find the number (new addend) to add to the 10.

In other words, when we use the right (10) end of the A scale, we add 10 to the number we read on the B scale that is directly below the second addend, found on the A scale. Do you understand? Try your hand at the examples in SET VII.

SET VII

Work these examples on your sliding rule. First be sure to ask yourself whether the example will involve carrying. If it does, use the **right** end of the A scale instead of the left end.

a. $7 + 5$

b. $4 + 8$

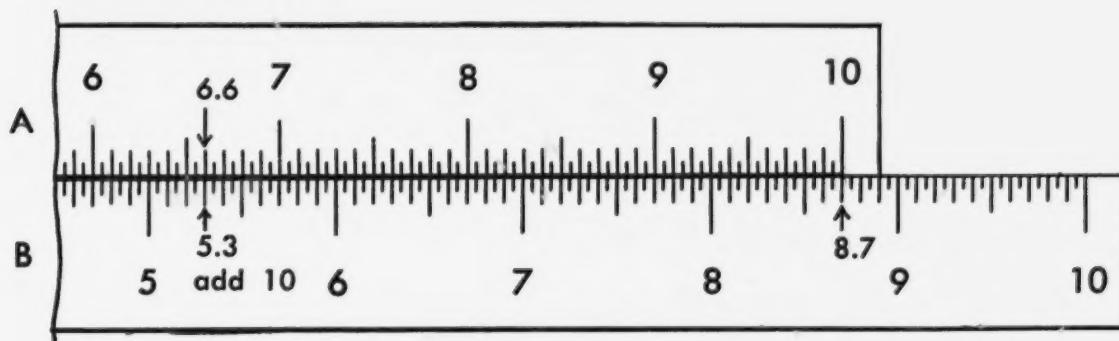
c. $3 + 6$

Check your thinking on page 61.

Two-place and even three-place numbers can be added in the same way as single digits. Try this example: $87 + 66$. First, we know we have to decide which index to use. Will 10 be right? Remember we need to divide **both** addends by 10. $87 \div 10 = 8.7$ and $66 \div 10 = 6.6$. Now locate the first addend, 8.7, on the B scale. Place the 10 end of the A scale directly above the 8.7, then locate 6.6 on the A scale. Read the number on the B scale that is directly below the 6.6 on the A scale. Is it 5.3? Since we used the right end of the scale we must add 10 to the 5.3, and this equals 15.3. We also used an index of 10 so now we must multiply 15.3 by 10 to get our answer. Can we do this by simply shifting the decimal point one place to the right? The sum of 87 and 66 is 153, right? You can check this by adding with pencil and paper.

Figure K below shows how you add $87 + 66$ on your sliding rule. Did you locate each numeral correctly?

Figure K



Try adding $45 + 36$ on your sliding rule without referring to this booklet. Then check with pencil and paper to see if you are correct. When you feel sure about how to do this, do SET VIII.

SET VIII

- | | |
|------------------|--------------------|
| a. $39 + 76 =$ | b. $4.7 + 3.9 =$ |
| c. $28 + 93 =$ | d. $67 + 59 =$ |
| e. $.68 + .93 =$ | f. $850 + 470 =$ |
| g. $380 + 950 =$ | h. $255 + 945 =$ |
| i. $705 + 850 =$ | j. $9.05 + 8.85 =$ |

Check your thinking on page 61.

You have already learned that subtraction is the opposite or undoing of addition. Let's look again at our first simple example, $3 + 5 = 8$ (on page 14), and "undo" it by subtraction. $8 - 5 = ?$ Can you figure out now to do this on the sliding rule? Which scale do you start with? Yes, the B scale. Which number do you locate on it? Yes, the minuend (8). When you locate the subtrahend (5) on the A scale, what will you do? Yes, you'll slide the A scale so that the 5 on it is directly over the 8 on the B scale. Now, where will you find the difference between these two numbers? Look at the sliding rule. Where is the answer (3)? Yes, it is shown on the B scale directly below the left end of the A scale.

You can also subtract two- and three-place numbers on the rule, including problems that require borrowing (regrouping). Do the example $37 - 18$ on your rule. Since you can't find 37 and 18 on the rule, you will need to choose an index. Will an index of 10 work? $37 \div 10 = ?$ $18 \div 10 = ?$ Can you locate 3.7 on the B scale and 1.8 on the A scale? Can you complete the subtraction on your own? Try it and then check your thinking with the following steps.

- Step 1** Did you place the 1.8 on the A scale directly above the 3.7 on the B scale?
- Step 2** Did you read the difference on the B scale directly below the left (zero) end of the A scale? (Note: You may have read the difference by counting back on the A scale to the right (10) end of the B scale. This will work but it is harder to see the answer immediately.)
- Step 3** Did you find that the difference was 1.9? Does $3.7 - 1.8 = 1.9$?
- Step 4** Did you remember that you had to multiply 1.9 by 10 to get 19 as an answer since you used an index of 10?

Try subtracting $56 - 39$ without referring to the above steps. Then check your work by subtracting mentally or with pencil and paper. When you are sure of yourself, try SET IX. Remember to use the same index for all the numbers in any one example.

SET IX

- | | |
|--------------------|-------------------|
| a. $87 - 45 =$ | b. $9.3 - 2.8 =$ |
| c. $178 - 86 =$ | d. $59 - 34 =$ |
| e. $6.1 - 2.9 =$ | f. $705 - 635 =$ |
| g. $635 - 285 =$ | h. $145 - 97 =$ |
| i. $3.85 - 2.65 =$ | j. $1.90 - .85 =$ |

In Examples c and h notice that you need to use an index of 100, so 86 will be .86 or less than 1, and 97 will be .97 or less than 1 on the sliding rule. Check your thinking on page 62.

PART II

Scientists, Symbols, and Shorthand

Mathematical Shorthand

Have you noticed in your reading of science materials, newspapers, and magazines how many extremely large and extremely small numbers you find? How awkward it would be to multiply and divide with these long numbers, even though they are usually rounded! But mathematicians have met this problem by developing a mathematical shorthand. For example, the earth travels almost 600,000,000 (six hundred million) miles around the sun each year. The astronomer would write this number as 6×10^8 . He would read this numeral as "six times ten to the **eighth** power." This mathematical shorthand way of writing numerals is called the standard form of scientific notation. It is often shortened to **scientific notation**.

Would you like to learn to write numerals in the form of scientific notation? Scientific notation is basic to understanding and using the slide rule, so let's see how easy it is to learn this kind of mathematical shorthand.

We have discussed earlier some ways we can take numbers apart and put them together again. Another way we can do this is by breaking a number down into its **factors**. The factors of a number are the numbers which, when multiplied together, give the original number (or product). For example, 2 and 3 are the factors of 6 because when we multiply 2 and 3 the product is 6. Does it matter in which order we multiply 2 and 3? $2 \times 3 = 6$ and $3 \times 2 = 6$; the product is the same.

Now, can a product have more than one pair of factors? Can you think of three numbers which when multiplied together give the product 8? We know that $2 \times 4 = 8$. Now, can you break either factor (the 2 or the 4) down into two factors? Yes, the 4 can be broken down into 2×2 . Thus $8 = 2 \times (2 \times 2)$. This is as far as we can break down 8

into its factors, so the three 2's are called the **prime** factors of 8. We couldn't break the 2's down, because 2 is a **prime** number; that is, its only factors are itself and 1.

Can you break 24 down into its prime factors? We know that $2 \times 12 = 24$ and that, since 2 is a prime number, the 2 cannot be broken down any further. How about the 12? $2 \times 6 = 12$. Can 6 be broken down any further? Yes, $2 \times 3 = 6$. Can we go any further? No, neither 2 nor 3 can be broken down any more since they are both prime numbers. What factors of 24 do we now have? $2 \times 12 = 24$; $12 = 2 \times 6$, and $6 = 2 \times 3$; so $24 = 2 \times 2 \times 2 \times 3$.

Is this right? Multiply the factors to be sure. Does it matter in which order you multiply? Does $3 \times 2 \times 2 \times 2 = 2 \times 2 \times 2 \times 3$? Keep this principle in mind so that later when you multiply such factors you can be confident that the order in which you put your factors will not change your answer.

Start with the factors of 8 and 3 and find the prime factors of 24. Do you still get the factors of 2, 2, 2, and 3? Try breaking down the factors 4 and 6 to find the prime factors of 24. Are they still 2, 2, 2, and 3? If you don't get the same prime factors each time, check your work again because a given number can be broken down into only one set of prime factors.

You noticed that when we broke 8 down into its prime factors, 2 was the only factor and it was used three times ($8 = 2 \times 2 \times 2$). When a number, such as the 2 in this case, is the only factor used, it is called a **base**. We can express $2 \times 2 \times 2$ in the form 2^3 . The 3 is called an **exponent**. The exponent tells us how many times the base is used as a factor. $2^3 = 2 \times 2 \times 2 = 8$.

We can read 2^3 as 2 to the third power. How would you read 2^2 ? Yes, 2 to the second power. $2^2 =$ what number? That's right, $2^2 = 2 \times 2 = 4$. How would you read 2^4 ? What number does 2^4 stand for? $2 \times 2 \times 2 \times 2 = 16$.

Of course, other numbers than 2 can be used as a base. For example, $3^3 = 27$. What number is used as a base here? Yes, 3; this is true because $3 \times 3 \times 3 = 27$. What number is used as a base in 5^3 ? Yes, 5. What about 10^2 ? Yes, here 10 is the base. Even though 10 is not a prime number, it is a base.

Why do you think we often use 10 as a base? Let's try expressing some numbers using base 10 and exponents. How would you express 100? How many times is 10 used as a factor in 100? Yes, 2; so $100 = 10^2$. Now can you express 1000 ($10 \times 10 \times 10$) using 10 and an exponent?

To be sure you understand, study the following table, which uses the base 10. Copy and complete it. Then check your thinking on page 62.

$$\begin{aligned} 10^1 &= 10 \\ 10^2 &= 10 \times 10 = 100 \\ 10^3 &= 10 \times 10 \times 10 = \\ 10^4 &= \\ 10^5 &= \\ 10^9 &= \end{aligned}$$

When your table is complete and correct, study it again. Do you see any relationship between the exponent in 10^2 and the number of zeros in 100? Do you see the same relationship between the number of zeros in 1000 and the exponent in 10^3 ? Is this true in each case? Yes, the exponent always indicates the number of zeros following the digit 1.

Do you remember about multiplying a number by 10? Study the set of examples below to see what happens.

1	10	100	1000	10,000
$\times 10$	$\times 10$	$\times 10$	$\times 10$	$\times 10$
10	100	1000	10,000	100,000

Notice that there is a place—in this case a 0—for each time that 10 is used as a factor. Knowing this makes it easy for you to change a number written with an exponent into a number as we usually write it.

How would you make the change in the other direction—that is, from a numeral the way we usually write it to the form that uses 10 and an exponent? Let's try with 10,000.

In 10,000 you know that 10 has been used as a factor 4 times because there are 4 zeros in the numeral. Since you know that division is the opposite of multiplication, you can prove that $10,000 = 10^4$ by doing repeated divisions of 10,000, etc., by 10, as shown below.

1,000.	100.	10.	1.	.1	.01
$10 \overline{) 10,000.}$	$10 \overline{) 1000.}$	$10 \overline{) 100.}$	$10 \overline{) 10.}$	$10 \overline{) 1.}$	$10 \overline{) .1}$

This set of examples also reminds us of a short way of dividing by 10. We merely shift the decimal point to the left one place each time we divide by 10. Keeping this in mind will be helpful in changing numbers from the way we usually write them to the way they are expressed in scientific notation.

For example, the planet Saturn is almost 1,000,000,000 miles from the sun. To write this number in scientific notation we shift the decimal point 9 places to the left, which gives the same result as if we had done 9 repeated divisions by 10 (1.000000000). We now have the single digit (1) at the left of the decimal point. To keep the original value (1,000,000,000) we must compensate by indicating that the single digit is to be multiplied by the proper power of 10. What is this power? Yes, 10^9 , or 1,000,000,000, because $1 \times 10^9 = 1,000,000,000$. Therefore 1,000,000,000 in scientific notation is 1×10^9 .

You notice that we refer to both 1,000,000,000 and 10^9 as a "power of 10." We can do this because 10^2 , 10^3 , 10^4 , . . . are merely shorter or more convenient ways of expressing 100 (10×10); 1000 ($10 \times 10 \times 10$); 10,000 ($10 \times 10 \times 10 \times 10$); . . . A power of 10 expressed with an exponent (the exponential form) shows at a glance the relative size of a number because the exponent tells how many times 10 is used as a factor. The exponential form also helps in keeping track of the decimal point in a series of multiplications or divisions.

Now let's look again at the two numbers we have expressed in scientific notation:

$6 \times 10^8 =$ the approximate number of miles the earth travels around the sun in one year.

$1 \times 10^9 =$ the approximate distance in miles the planet Saturn is from the sun.

Do you notice that in each of these cases we have used a number consisting of a **single digit** (1, 6) followed by zeros? To change the numbers to scientific notation all we had to do was to show the single digit multiplied by a power of 10 written with an exponent.

But now suppose you had to deal with the number stating the distance from the earth to the sun: 93,000,000 miles (approximately). You know that 93,000,000 can be expressed as

93×10^6 because there are 6 zeros:

$93 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$; 10 has been used as a factor 6 times.

However, 93×10^6 is still not expressed in scientific notation because in scientific notation there must be only one digit (other than 0) to the left of the decimal point. We still have the problem of expressing the 93 with only one digit to the left of the decimal point, but with the original value of 93 unchanged.

You can probably see right away that all we need to do is to divide by 10 (shift the decimal point one place to the left) to get 9.3, and compensate by showing the 9.3 as multiplied once by 10. We now have 9.3×10^1 , which means that we have used 10 as a factor one more time:

$(9.3 \times 10) \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$; 10 has been used as a factor 7 times all together.

Therefore 93,000,000 expressed in scientific notation is 9.3×10^7 .

Using what you have learned, how can you express 3,700,000,000—the distance in miles to Pluto from the sun—in scientific notation? Try this by yourself, then read the following explanation.

Did you think: I know that 3,700,000,000 can be expressed as 37×10^8 . But in scientific notation I can have only one digit (not 0) to the left of the decimal point, so I must divide again by 10 to get 3.7×10^1 . I have divided by 10 nine times; therefore in scientific notation 3,700,000,000 is expressed as 3.7×10^9 .

Now try your hand at expressing 157 in scientific notation. This may at first seem puzzling to you since there are no zeros in this numeral. But you remember that our decimal place-value system, since it is based on 10, permits us to express any number in terms of 10. What shall we do, then, with 157? We can divide by 10. Let's shift the decimal point one place to the left: $157 = 15.7 \times 10$. You can see that we need to divide by 10 again, so we move the decimal point one more place to the left, giving us $157 = 1.57 \times 10 \times 10$ or 1.57×10^2 . Therefore 157 in scientific notation is 1.57×10^2 .

value of the fraction does not change. Therefore, we can change

$\frac{36}{1000}$ (.036) to hundredths by dividing both 36 (the numerator)

and 1000 (the denominator) by 10. Using the short way to divide by 10 we move the decimal point (which is at the right of the ones' place even though it isn't written) one place to the left. So 36 becomes 3.6 and 1000 becomes 100.

$\frac{36 \div 10}{1000 \div 10} = \frac{3.6}{100}$; this is read "3 point six hundredths."

$\frac{3.6}{100}$ can be changed to $\frac{36}{1000}$ by multiplying both numerator

and denominator by 10: $\frac{3.6 \times 10}{100 \times 10} = \frac{36}{1000}$. Now use the

principle of compensation to change .25 ($\frac{25}{100}$) to tenths writ-

ten as a common fraction. Do you divide each term by 10?

$\frac{25 \div 10}{100 \div 10} = \frac{2.5}{10}$, doesn't it? How do you read this fraction?

To express a decimal fraction in scientific notation, in the way that we are showing you, we must first write it as a common fraction. Then we change the numerator so that there is a single digit to the left of the decimal point. We make this change by dividing the numerator by 10 or a power of 10; then compensate by making the same division in the denominator. Let's do these steps with the decimal fractions a, b, and c.

$$\text{a. } .00365 \text{ (or } \frac{365}{100,000} \text{)} = \frac{3.65}{?}$$

By what did we divide the numerator 365 to get 3.65? Yes, by 100. Therefore we have to divide the denominator by 100, which gives us a new denominator of 1000. Does $.065 = \frac{6.5}{100}$?

$$\text{b. } .065 \text{ (or } \frac{65}{1000} \text{)} = \frac{6.5}{?}. \text{ By what number will you divide}$$

the denominator? Yes, by 10. So $.065 = \frac{6.5}{100}$.

c. $.000346 \left(\frac{346}{1,000,000} \right) = \frac{3.46}{?}$ By what number will you divide? 10? 100? Yes, by 100, or by 10 and then by 10 again. So, $.000346 = \frac{3.46}{10,000}$.

In each fraction, a, b, and c above, the numerator of the fraction now fulfills the requirements for scientific notation. Only one step is left, that is, to express each denominator using 10 and an exponent. That will be easy, won't it?

$\frac{3.65}{1000} = \frac{3.65}{(10 \times 10 \times 10)} = \frac{3.65}{10^3}$. Can you change the denominator of $\frac{6.5}{100}$ to a numeral with an exponent? $100 = 10 \times 10$, and $10 \times 10 = 10^2$, so $\frac{6.5}{100} = \frac{6.5}{10^2}$. What about $\frac{3.46}{10,000}$? How many times is 10 used as a factor in 10,000? Yes, 4 times, so $\frac{3.46}{10,000} = \frac{3.46}{10^4}$, doesn't it?

When you are thoroughly familiar with expressing decimal fractions in scientific notation, try SET XII to be sure you understand this form of expressing numbers.

SET XII

Write the following numerals in scientific notation.

- | | | | |
|-----------|---------------|------------|------------|
| a. .0647 | b. .000398 | c. .247 | d. .000005 |
| e. .00349 | f. 35,000,000 | g. 964,000 | h. .00539 |
| | i. 94.347 | j. 903.45 | |

Now, change the rest of these back from scientific notation. For example, to help you get back to a decimal fraction, look

at these steps: $\frac{3.2}{10^2} = \frac{3.2}{100} = \frac{32}{1000} = .032$.

- | | | | |
|-------------------------|-------------------------|-----------------------|-------------------------|
| k. $\frac{6.7}{10^2}$ | l. $\frac{3.943}{10^3}$ | m. 5.43×10^3 | n. $\frac{8.003}{10^2}$ |
| o. $\frac{3.958}{10^3}$ | p. $\frac{6.894}{10^5}$ | q. 1.45×10^7 | r. $\frac{2.593}{10^3}$ |
| s. $\frac{9.993}{10^1}$ | t. 8.395×10^2 | | |

Check your thinking on pages 64 and 65.

PART III

Estimating Answers on the Slide Rule

A key skill in multiplying and dividing with a slide rule is locating the decimal point; that is, determining the number of places in your answer. If we were to multiply 368×243 with a slide rule, we would read only the three digits **894**. Obviously the product is larger than 894, but is it 8940, 89,400 or 894,000? We need to estimate a reasonable answer first.

You have probably already used estimation in many ways as you worked in arithmetic; however, you will need to learn more about it to estimate your answers on the slide rule. You will need to combine your understanding of scientific notation with your knowledge of rounding numbers. For example, if we expressed the multiplier, 368, in the example above in scientific notation, it would be 3.68×10^2 . We now want to round 3.68 to a **single digit**. Since the first digit to the right of the decimal point is 5 or more, we increase the digit to the left of the decimal point by 1, so 3.68 rounds to 4. For purposes of estimation we can now think of the multiplier as 4×10^2 .

The multiplicand, 243, in scientific notation is 2.43×10^2 . We must round 2.43 to a single digit. We again look at the digit directly to the right of the decimal point; it is less than 5, so 2.43 is rounded to 2. We can think of our multiplicand as 2×10^2 .

Our multiplication example is now approximately $(4 \times 10^2) \times (2 \times 10^2)$. We can also rearrange the factors like this: $(4 \times 2) \times (10^2 \times 10^2)$. This is possible because when we multiply, changing the order of the factors does not change the product. We know how to multiply 4×2 , but how do we multiply numbers expressed as 10 and an exponent? You will recall that multiplication is a short way of adding groups of equal size. We can use this understanding to learn to multiply numbers which are expressed with the same base, which in this case is 10. Let's see why this is true. You know that $10^2 =$

10×10 . So in our example, $(4 \times 10^2) \times (2 \times 10^2)$, we have $(10 \times 10) \times (10 \times 10)$. How many times has 10 been used as a factor? Yes, 4 times, so $(10 \times 10) \times (10 \times 10) = 10^4$, or $10^2 \times 10^2 = 10^4$. Do you see that to multiply numbers expressed as 10 and an exponent we merely **add** the exponents? This makes it easy, doesn't it? Let's multiply $10^4 \times 10^3$. Will your answer be 10^7 (10,000,000)? You can check by multiplying: $(10 \times 10 \times 10 \times 10) \times (10 \times 10 \times 10)$.

Use the examples in SET XIII to practice multiplying numbers expressed as 10 and an exponent.

SET XIII

a. $10^2 \times 10^3 =$

b. $10^3 \times 10^5 =$

c. $10^5 \times 10^7 =$

d. $10^8 \times 10^3 =$

Check your thinking on page 65.

Let's continue with the example we began on page 31. It is now $4 \times 2 \times 10^4$ or 8×10^4 . If the multiplication is completed, we get an estimated product of 80,000. Let's compare this with the answer **894** we read on the slide rule. We can see that our product is approximately 89,400 since this is closer to the estimate, 80,000, than is 894,000 or 8940.

Let's use the following steps to estimate the answer to another multiplication example. As read on the slide rule the first three digits of the product 87.9×3210 are **282**.

Step 1 We change the multiplicand and multiplier to scientific notation: $87.9 = 8.79 \times 10^1$; $3210 = 3.210 \times 10^3$, right?

Step 2 We round 8.79 to 9 and 3.210 to 3.

Step 3 Our approximate example is now $(9 \times 10^1) \times (3 \times 10^3)$, or $9 \times 3 \times 10^1 \times 10^3$.

Step 4 We complete the multiplication: $9 \times 3 = 27$, $10^1 \times 10^3 = 10^4$; $27 \times 10^4 = 270,000$.

Step 5 We now compare our estimated product with the answer we read on the slide rule. 282,000 is closer to 270,000 than is either 28,200 or 2,820,000, so our approximate product is 282,000.

SET XIV below gives some examples with the products shown as they would be read on the slide rule. We will use an arrow ($\longrightarrow$) to indicate "the numerals we read on the slide rule." By estimation find how many places there will be in your answer; that is, locate the decimal point.

SET XIV

a. $392 \times 483 \longrightarrow 189$

b. $88.3 \times 570,000 \longrightarrow 503$

c. $8240 \times 8.39 \longrightarrow 691$

d. $45,000 \times 3.14 \longrightarrow 141$

Check your answers on page 66.

In estimating in division we also use scientific notation to express the divisor and the dividend and then round the numbers. After that we are ready to divide the numbers expressed as 10 and an exponent. Consider the example $10^5 \div 10^3$. Think about the relationships between the four processes. How is division related to multiplication? We know that it is the inverse, or undoing, of multiplication. And we have just recalled that multiplication is a short way of adding equal addends. Now, how is division related to subtraction? It's a short way of subtracting groups of equal size, isn't it? It makes sense, then, that if we add exponents when we multiply numbers expressed as 10 and an exponent, we should subtract exponents when we divide such numbers.

We can see this more clearly by looking at a division example written in the form of a common fraction, as below:

$$10^5 \div 10^3 = \frac{10^5}{10^3} = \frac{10 \times 10 \times 10 \times 10 \times 10}{10 \times 10 \times 10}$$

Using the principle of compensation, we can cancel all units of

$\frac{10}{10}$ in the example. $\frac{\cancel{10} \times \cancel{10} \times \cancel{10} \times 10 \times 10}{\cancel{10} \times \cancel{10} \times \cancel{10}}$

What do we have left? 10×10 ? Yes, or 10^2 . In other words, we subtract the exponent 3 of the divisor from the exponent 5 of the dividend and the remainder is 2, which is the exponent of the quotient; therefore 10^2 is our quotient.

If the dividend is smaller than the divisor, for example, $10^3 \div 10^5$, it will look like this: $\frac{10 \times 10 \times 10}{10 \times 10 \times 10 \times 10 \times 10} = \frac{1}{10^2}$ or $\frac{1}{10^2}$.

Notice that in this example the difference between the exponents in the dividend and divisor is still used, but since the divisor is larger, the difference is left in the denominator and is used as a divisor. Use the examples in SET XV to practice dividing numbers expressed in exponential form.

SET XV

a. $10^7 \div 10^2 =$ b. $10^9 \div 10^3 =$ c. $10^2 \div 10^5 =$

Check your thinking on page 66.

Let's use what we have learned to estimate an answer in division. We can try the example $77,500 \div 212 \longrightarrow 366$.

Step 1 Express the divisor and dividend in scientific notation: $77,500 = 7.75 \times 10^4$; $212 = 2.12 \times 10^2$.

Step 2 Round the first number of each: 7.75 is rounded to 8, and 2.12 is rounded to 2.

Step 3 Write the approximate example:
 $(8 \times 10^4) \div (2 \times 10^2) =$
 We know how to divide whole numbers ($8 \div 2$ or $\frac{8}{2}$) and we know how to divide numbers expressed as 10 and an exponent, but these divisions are not done the same way. Dividing is easier if we rewrite the example in common fraction form. (Any division can be written as a common fraction.)

Step 4 Rewrite the approximate example:
 The dividend is the numerator $\frac{8 \times 10^4}{2 \times 10^2}$
 The divisor is the denominator
 Now change to this form: $\frac{8}{2} \times \frac{10^4}{10^2}$
 You can either multiply or divide first, but if you divide first computation is easier.

- Step 5** Complete the computation to find the estimated quotient: $4 \times 10^2 = 400$.
- Step 6** Compare the estimate with the answer as read on the slide rule. 366 is the quotient since it is closest to the estimate, 400 (3660 would be too large and 36.6 would be too small).

Study the following example, $.832 \div 59.4 \rightarrow 140$, until you become familiar with the steps in estimating an answer when dividing on the slide rule.

- Step 1** Write both divisor and dividend in scientific notation: $.832 = 8.32 \times \frac{1}{10^1}$; $59.4 = 5.94 \times 10^1$

- Step 2** Round the first number of the divisor and the dividend: 8.32 to 8 and 5.94 to 6.

- Step 3** Write the approximate example:

$$(8 \times \frac{1}{10^1}) \div (6 \times 10^1) =$$

- Step 4** Rewrite the example as a common fraction:

$$\frac{8 \times \frac{1}{10^1}}{6 \times 10^1} = \frac{8}{6} \times \frac{1}{10^1} =$$

- Step 5** Complete the computation doing the division first:

a. $\frac{8}{6} = 1\frac{2}{6} = 1\frac{1}{3}$ or approximately 1;

b. $\frac{1}{10^1}$ can now be written as a division example

in common fractions: $\frac{1}{10^1} \div 10^1 = \frac{1}{10^1} \div \frac{10^1}{1}$.

To complete this division, we know we must invert the divisor and multiply:

$$\frac{1}{10^1} \times \frac{1}{10^1} = \frac{1}{10^2} = \frac{1}{100} = .01$$

- Step 6** Compare the estimated quotient with the digits read on the slide rule. .0140 is the approximate quotient since it is closest to .01, right?

There is still another type of division example, that is, one that has a divisor with a value less than 1, such as $365 \div .024 \rightarrow 152$. Examples such as this are more difficult, but you may want to try to do them. If so, study the following steps in estimating the answer to this example: $365 \div .024 \rightarrow 152$.

Step 1 Write divisor and dividend in scientific notation:

$$365 = 3.65 \times 10^2; .024 = 2.4 \times \frac{1}{10^2}.$$

Step 2 Round: 3.65 to 4; 2.4 to 2.

Step 3 Write approximate example:

$$(4 \times 10^2) \div (2 \times \frac{1}{10^2}) =$$

Step 4 Rewrite the example: $\frac{4 \times 10^2}{2 \times \frac{1}{10^2}} = \frac{4}{2} \times \frac{10^2}{\frac{1}{10^2}}$

Step 5 Complete the computation: $\frac{4}{2} = 2$;

$$\frac{10^2}{\frac{1}{10^2}} = \frac{10^2}{1} \div \frac{1}{10^2} = \frac{10^2}{1} \times \frac{10^2}{1} = \frac{10^4}{1} = 10^4;$$

$$2 \times 10^4 = 20,000.$$

Step 6 Compare the estimated quotient with the digits as read on the slide rule. Of the three possible answers—152,000, 15,200, and 1520—the closest to the estimate (20,000) is 15,200.

SET XVI will give you an opportunity to try out estimation. Eliminate whatever steps you can by doing mental arithmetic. You can probably combine Steps 1 and 2 immediately.

The answer is given for each of the following examples as read from the slide rule. Copy each example and place the decimal point correctly in the quotient by estimation.

SET XVI

a. $36,700 \times 578 \rightarrow 212$

b. $685 \div .342 \rightarrow 200$

c. $.359 \times 48,000 \rightarrow 172$

d. $468 \times .00583 \rightarrow 273$

e. $938 \div 26,500 \rightarrow 354$

Check your answers on pages 67 and 68.

If you feel you need further practice in estimating in either multiplication or division, try SET XVII.

SET XVII

- | | |
|---|---|
| a. $.0876 \div .324 \rightarrow 270$ | b. $348 \times .683 \rightarrow 238$ |
| c. $.0056 \times .038 \rightarrow 213$ | d. $.369 \div .072 \rightarrow 513$ |
| e. $56,700 \div 2380 \rightarrow 238$ | f. $.695 \times 4590 \rightarrow 319$ |
| g. $.078 \times 3460 \rightarrow 27$ | h. $9.26 \times .045 \rightarrow 417$ |
| i. $34,500 \times 68.300 \rightarrow 236$ | j. $.00824 \times .036 \rightarrow 296$ |

Check your answers on page 69.

Logarithms and the Slide Rule

The secret of the slide rule is in the special way that its scales are graduated. What is the basis of these special scales? Have you ever heard of logarithms?

Just what is a logarithm and how do we use logarithmic scales to speed up our computations? A logarithm, or "log" as it is often called, is an exponent of a particular number which is called the base. It is the power to which the base must be raised to equal the number. For example, the log of 100 to the base 10 is 2 because 10 must be raised to the power 2 to equal 100; $10 \times 10 = 10^2 = 100$. Study the following table of logarithms using the base 3.

	No.	log		No.	log
$3^1 =$	3	1	$3^6 =$	729	6
$3^2 =$	9	2	$3^7 =$	2,187	7
$3^3 =$	27	3	$3^8 =$	6,561	8
$3^4 =$	81	4	$3^9 =$	19,683	9
$3^5 =$	243	5	$3^{10} =$	59,049	10

As you study this table you see that the log of these numbers is the exponent of the base 3; that is, the log of 81 is 4 because 3, the base, must be raised to the power 4 to equal 81. $3 \times 3 \times 3 \times 3 = 81$. What is the log of 729 to the base 3? Yes, 6, because $3^6 (3 \times 3 \times 3 \times 3 \times 3 \times 3) = 729$.

Logarithm tables can be made using any base except 1, but the most common and most useful table has 10 as a base. Can you think why this is true? Your teacher will supply you with a base-10 logarithm table for computing with three-place numerals.

But first we must learn to read the log table. Suppose we want to find the log of 574. Looking at the left side of the table in the column headed No. (which stands for number) you notice that there are no three-place numerals; in fact, the numerals under No. stand only for the numbers between 1 and 10. (The decimal point is not always shown on a log table.) Therefore we must change 574 to scientific notation to reduce it to a number that is between 1 and 10. In scientific notation $574 = 5.74 \times 10^2$. We can find the first two digits, 5.7, in the left-hand column headed No. For the third digit of 5.74, we move across to the column headed 4, and find the log .7589.

Figure L TABLE OF LOGARITHMS (Base-10)

No.	0	1	2	3	4	5	6	7	8	9
5.5	.7404	.7412	.7419	.7427	.7435	.7443	.7451	.7459	.7466	.7474
5.6	.7482	.7490	.7497	.7505	.7513	.7520	.7528	.7536	.7543	.7551
5.7	.7559	.7566	.7574	.7582	.7589	.7597	.7604	.7612	.7619	.7627
5.8	.7634	.7642	.7649	.7657	.7664	.7672	.7679	.7686	.7694	.7701
5.9	.7709	.7716	.7723	.7731	.7738	.7745	.7752	.7760	.7767	.7774
6.0	.7782	.7789	.7796	.7803	.7810	.7818	.7825	.7832	.7839	.7846
6.1	.7853	.7860	.7868	.7875	.7882	.7889	.7896	.7903	.7910	.7917
6.2	.7924	.7931	.7938	.7945	.7952	.7959	.7966	.7973	.7980	.7987
6.3	.7993	.8000	.8007	.8014	.8021	.8028	.8035	.8041	.8048	.8055
6.4	.8062	.8069	.8075	.8082	.8089	.8096	.8102	.8109	.8116	.8122

For practice find the log of 6.32. Locate the first two digits, 6.3, in the column headed No. Then move across the row from 6.3 to the column headed 2 to find the place where this row and the column meet (or intersect). The log in this place is the one you are looking for (.8007).

Now find the logs of the numerals in SET XVIII.

SET XVIII (Check your answers on page 69.)

- a. 1.44

b. 3.03

c. 6.05

d. 5.49
- e. 7.38

f. 3.60

g. 5.87

h. 9.40
- i. 6.39

j. 5.43

2.5	.3979	.3997	.4014	.4031	.4048	.4065	.4082	.4099	.4116	.4133
2.6	.4150	.4166	.4183	.4200	.4216	.4232	.4249	.4265	.4281	.4298
2.7	.4314	.4330	.4346	.4362	.4378	.4393	.4409	.4425	.4440	.4456
2.8	.4472	.4487	.4502	.4518	.4533	.4548	.4564	.4579	.4594	.4609
2.9	.4624	.4639	.4654	.4669	.4683	.4698	.4713	.4728	.4742	.4757
3.0	.4771	.4786	.4800	.4814	.4829	.4843	.4857	.4871	.4886	.4900
3.1	.4914	.4928	.4942	.4955	.4969	.4983	.4997	.5011	.5024	.5038
3.2	.5051	.5065	.5079	.5092	.5105	.5119	.5132	.5145	.5159	.5172
3.3	.5185	.5198	.5211	.5224	.5237	.5250	.5263	.5276	.5289	.5302
3.4	.5315	.5328	.5340	.5353	.5366	.5378	.5391	.5403	.5416	.5428

When you feel confident about finding logs of three-place numbers, you are ready to learn how to do the opposite operation—that is, find a number when you know its log. You will need to do this when you use logs for multiplying and dividing. Let's try an example to see how.

Suppose we have the log .5250. What number corresponds to this log? Read down the column headed 0 (notice that the logs get larger) until you find the first log in this column that is larger than .5250, the log you are looking for. Move back up from .5315 one row to .5185 and then look across to the right to find .5250. This log is in the row starting with 3.3 under No. and in the column headed 5; therefore 3.35 is the number whose log is .5250.

Many times you may have a log which does not appear directly in the table. In this booklet we will use a method of approximation to decide what number such a log represents. For example, suppose we wanted to find the number whose log is .7821. We do not find .7821 listed in the table, but we do find the next smaller log, .7818, and the next larger log, .7825. The difference between these listed logs is .0007.

Think of the difference (.0007) as 7 steps on a number scale. From .7818 to .7821 is 3 steps; from .7821 to .7825 is 4 steps. We can see that our log, .7821, is closer to .7818 than to .7825, so for our purposes we will select as the approximate answer the number whose log is .7818. This number is 6.05.

Now suppose we wanted to find the number whose log is .7822. Again thinking of steps along a number scale, we see that from .7818 to .7822 is 4 steps, and from .7822 to .7825 is 3 steps. In this case our log, .7822, is closer to the larger listed log, so we will select as the approximate answer the number whose log is .7825. This number is 6.06.

Review any parts of this section you feel unsure of, and when you are quite sure that you know how to find a number from its log, do SET XIX.

SET XIX

Use your table of logarithms to find the number for each log below. Check your thinking on page 69.

- | | |
|-----------------------------|-----------------------------|
| a. .0212 is the log of ____ | b. .2504 is the log of ____ |
| c. .6085 is the log of ____ | d. .8062 is the log of ____ |
| e. .9666 is the log of ____ | f. .9939 is the log of ____ |
| g. .7459 is the log of ____ | h. .6425 is the log of ____ |
| i. .8791 is the log of ____ | j. .6021 is the log of ____ |

Now that you can find the log of a number and find a number from its log, you are ready to use a log table to multiply and divide numbers. Let's see how easy it is to multiply 320×260 . First we write the multiplier (320) and the multiplicand (260) in scientific notation: $320 = 3.20 \times 10^2$; $260 = 2.60 \times 10^2$. Then we locate the logs of 3.20 and of 2.60. We find that .5051 is the log of 3.20 and .4150 is the log of 2.60.

To multiply, just add the logs of the numbers. Remember that the log of a number is the power to which the base 10 is raised to equal the number. When we add the two logs we are getting the power to which 10 is raised to equal the numbers multiplied together.

$$\begin{array}{r} \log 3.20 = .5051 \\ \log 2.60 = .4150 \\ \hline .9201 \end{array}$$

We look at the table to find the number which has a log equal to or near the sum of the logs (.9201). We find that .9201 is the log of 8.32. We can then place the decimal point in our approximate product by estimation as we learned earlier. We can round 3.20 to 3 and 2.60 to 3 and multiply: $(3 \times 3) \times (10^2 \times 10^2) = 9 \times 10^4 = 90,000$. Therefore our answer is 83,200 since this is closer to the estimated answer than is either 8320 or 832,000.

Practice using the log table to multiply by studying the steps in working the following three types of examples. (Notice the similarity between Example A and the one we just did. How are they alike? How are they different? Is placing the decimal point important?)

Example A $3.20 \times 2.60 = ?$

Step 1 Find the logs of the numbers. $\log 3.20 = .5051$
 $\log 2.60 = .4150$

Step 2 Add the logs to find their sum. $.9201$

Step 3 Find the number whose log is the sum. $.9201$ is the log of 8.32.
Therefore $3.20 \times 2.60 = 8.32$.

Estimating, you can readily see that you are multiplying approximately 3 (2.60 is almost 3) $\times$ 3. The approximate answer is 8.32, since it is closer to 9 than is .832 or 83.2.

Let's try another example.

Example B $4960 \times 82.7 = ?$

Step 1 Change to scientific notation. $4960 = 4.96 \times 10^3$
 $\times 82.7 = 8.27 \times 10^1$

Step 2 Find logs and add. $\log 4.96 = .6955$
Notice that the sum is $\log 8.27 = .9175$
greater than 1. Do you $(1).6130$
know why? Remember that 1 is the log of 10. We therefore know that the product is more than 10. Since we will place the decimal point by estimation, we can ignore the digit at the left of the decimal point.

Step 3 Find the number which has a log equal to or near the sum of the logs. $.6130$ is the approximate log of 4.10.

Step 4 Round 4.96 to 5 and 8.27 to 8. Complete the multiplication; then estimate for decimal point placement. $(5 \times 8) \times (10^3 \times 10^1) = 40 \times 10^4 = 400,000$; therefore the approximate answer is 410,000 since this is closer to 400,000 than is 41,000 or 4,100,000.

In Example C we will use the same steps as those we used in Examples A and B, but without explanation. Can you follow the steps? Try them now and see. You may want to review the section on exponents, pages 23 through 30, to remind you how we change .00596 to scientific notation.

Example C $.00596 \times 39.4 = ?$

Step 1 $.00596 = 5.96 \times \frac{1}{10^3}$

$$39.4 = 3.94 \times 10^1$$

Step 2 $\log 5.96 = .7752$
 $\log 3.94 = \frac{.5955}{(1).3707}$

Step 3 .3707 is the approximate log of 2.35.

Step 4 $(6 \times 4) \times (\frac{1}{10^3} \times 10^1) = 24 \times \frac{1}{10^2} = .24;$

therefore the answer is approximately .235 since .235 is closer to .24 (.240) than is .0235 or 2.35.

After you have carefully studied the preceding illustrations, try SET XX using logarithms. Check your thinking on page 69.

SET XX

a. $2.34 \times 3.63 =$

b. $6.38 \times 1.85 =$

c. $36.4 \times 289 =$

d. $348 \times 2.23 =$

e. $7.03 \times 5.40 =$

f. $4.94 \times 8.93 =$

g. $89.3 \times 494 =$

h. $6.49 \times 8.27 =$

i. $7.58 \times 396 =$

j. $.043 \times .647 =$

We have seen that logarithms make it easy to multiply once we know how to find the logs of numbers, add the logs, find the number whose log is the sum, and then estimate location of the decimal point. We use these same steps in dividing except that now we subtract the log of the divisor (denominator) from the log of the dividend (numerator). Study the steps in the following examples to see how we divide using logs.

Example A $8.34 \div 4.25 = ?$

Step 1 Find the logs and subtract.

$$\begin{array}{r} \log 8.34 = .9212 \\ \log 4.25 = \underline{.6284} \\ .2928 \end{array}$$

Step 2 Find the number whose log is the difference.

.2928 is the approximate log of 1.96.

Step 3 Round the divisor and dividend and divide to make an estimation. Place the decimal point by estimation.

$8 \div 4 = 2$; therefore the answer is approximately 1.96 since it is closer to 2 than is .196 or 19.6.

Example B $334,000 \div 848 = ?$

Step 1 Change to scientific notation.

$$\begin{array}{l} 334,000 = 3.34 \times 10^5 \\ 848 = 8.48 \times 10^2 \end{array}$$

Step 2 Find the logs and subtract. Notice that in order to allow us to subtract a 1 has been placed at the left of

$$\begin{array}{r} \log 3.34 = (1).5237 \\ \log 8.48 = \underline{.9284} \\ .5953 \end{array}$$

the decimal point in the minuend. This would correspond to multiplying the number by ten, but since we use estimation for placing the decimal point the 1 can be used here without further consideration. We will take care of this when we locate the decimal point by estimation.

Step 3 Find the number whose log is the difference.

.5953 is the approximate log of 3.94.

Step 4 Round the divisor and dividend and rewrite the example. Complete the computation.

$$\begin{array}{l} 3.34 \text{ rounds to } 3; \\ 8.48 \text{ rounds to } 8. \\ (3 \div 8) \times (10^5 \div 10^2) = \\ .4 \text{ (about)} \times 10^3 = 400. \end{array}$$

Step 5 Place the decimal point by estimation.

The answer is about 394 since 394 is closer to 400 than is 3940 or 39.4.

Now, let's consider one more example. Study it carefully to see if you understand the steps. If you have difficulty, review Examples A and B again.

Example C $3.98 \div .00843 = ?$

Step 1 $3.98 = 3.98$

$$.00843 = 8.43 \times \frac{1}{10^3}$$

Step 2 $\log 3.98 = (1).5999$ (using 1 to allow for
 $\log 8.43 = \frac{.9258}{.6741}$ subtracting)

Step 3 .6741 is the approximate log of 4.72.

Step 4 $(4 \div 8) \times (1 \div \frac{1}{10^3}) = .5 \times 10^3 = 500;$

therefore the answer is about 472.

If you are not sure you understand, go back and try again. However, if you think you really have the idea, go on to SET XXI and see how well you can apply what you have learned.

SET XXI

Use logs to solve the following examples.

a. $8.49 \div 3.48 =$

b. $7.95 \div 1.73 =$

c. $3.25 \div 5.34 =$

d. $4.75 \div 8.99 =$

e. $47.5 \div .899 =$

f. $381 \div 45.9 =$

g. $289 \div 348 =$

h. $.384 \div 90.6 =$

i. $.0528 \div 3.49 =$

j. $392 \div .00472 =$

Check your answers on page 70.

If you correctly completed most of the examples in SETS XX and XXI, you are ready for the next part of this booklet. You may want to review the steps that you have gone through so far. If you have any questions, do a little research and find the answers. Maybe you are not sure of yourself on some points. If so, take time out to make up some multiplication and division examples of your own and use logs to work them. Check the answers by doing the work in the regular way.

PART IV

Using a Slide Rule

Making a Slide Rule

What we have learned about number scales, sliding rules, and exponents, including logarithms, will help us understand how the slide rule works. Let's see how we can use what we know to make a simple slide rule out of the paper sliding rule we have used before. Follow these steps:

Step 1 Make a list of numbers from 1 through 10 in steps of .5. Use your logarithm table to find the log of each number in your list. Look under **No.** to find 1.0, 1.5, 2.0, . . . 9.5 and find the log directly to its right.

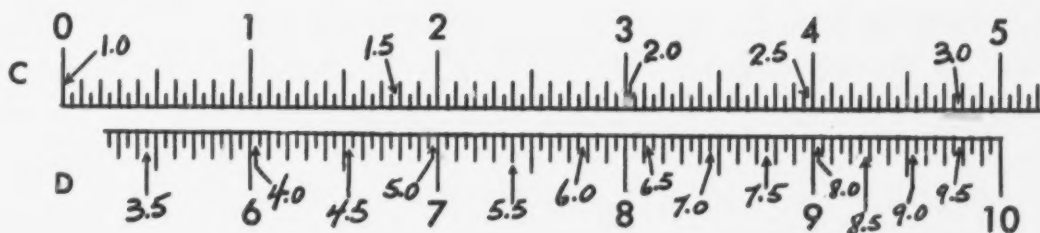
No.	log
1.0	.0000
1.5	.1761
2.0	.3010
9.5	.9777
log 10 is 1.0000	

Step 2 Find and mark on the sliding rule the position for each log in your list. Use an index of 10 so that the first digit of each log value is a whole number. For example, $\log 1.5 = .1761$; this becomes 1.761. We find 1.761 on the sliding rule between 1 and 2. (See Figure M. Notice that we use graduation marks to locate the first two digits, then estimate the location of the third digit. We ignore the fourth digit.)

Where did you locate the log of 1, which is .0000? That's right, at the extreme left end.

Step 3 Now label the marks with the numbers (1.0, 1.5, etc.) whose log values you have located on the rule. Label the top scale **C** and the bottom scale **D**, as a regular slide rule is labeled.

Figure M



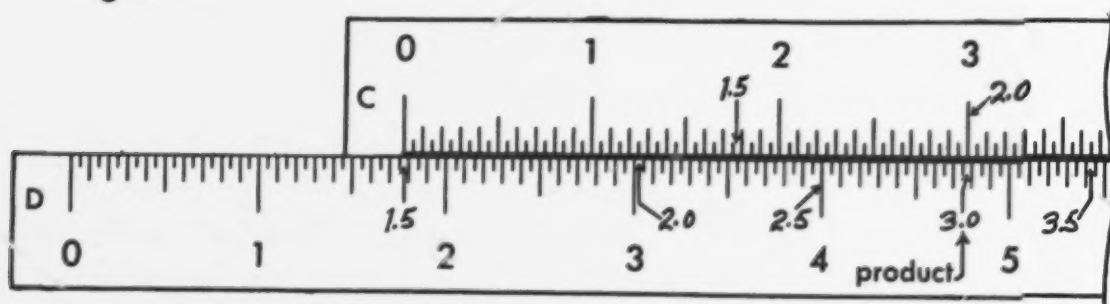
Look at the rule you have made. Remember that the log values we marked represent numbers from 1 through 10 in **equal** intervals of .5. But the distances on the scales between these log values are **not** equal. As you move to the right the distances which represent .5 get progressively smaller. Why? Because the slide rule is a **logarithmic** scale, one type of non-uniform scale.

On the slide rule multiplying and dividing numbers is easier than using the logarithm table because the slide rule does the work for us. The slide rule adds and subtracts the log values of numbers in the same way that the sliding rule adds and subtracts regular numbers. Let's try an example on the simple slide rule we have just made: $1.5 \times 2.0 = ?$

Using the marks you have labeled, first find 1.5 on the D scale. Place the left end of the C scale directly above this location. Now find 2.0 on the C scale and look directly below it on the D scale to find the product, 3.0.

How would you multiply 15×20 ? Can you see that the placement of the rule will be the same as in 1.5×2.0 ? The difference is in placing the decimal point in the product. You have already learned a way to do this by estimation. Figure N shows the placement of the rule for this example and also the positions of other log values you will need for SET XXII.

Figure N



Look at the placement of the rule in Figure N. Remembering that division is the undoing of multiplication, can you see how to do this example: $3.0 \div 2.0$? Where on the rule is the dividend? Where is the divisor? Can you see that you read the quotient on the D scale below the left end of the C scale?

Now try the examples in SET XXII using the slide rule you have made. Check your thinking on page 70.

SET XXII

a

1. $1.5 \times 3.0 =$
2. $3.5 \times 2.0 =$
3. $4.0 \times 1.5 =$
4. $6.0 \times 5.0 =$
5. $15 \times 6.0 =$

b

6. $7.5 \div 5.0 =$
7. $4.5 \div 1.5 =$
8. $900 \div 3.0 =$
9. $75 \div .25 =$
10. $.005 \div .25 =$

Reading the Slide Rule

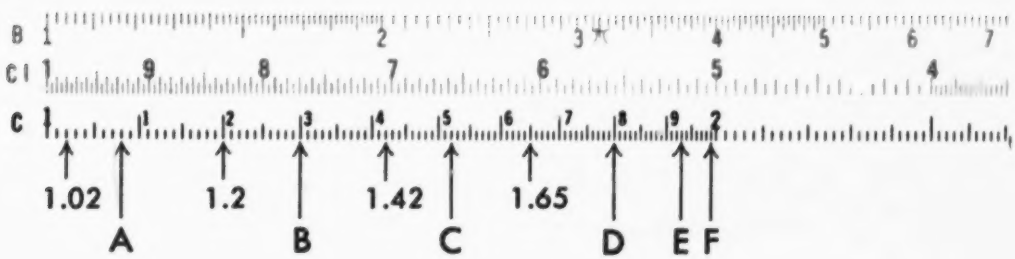
Congratulations! You are now ready to begin to use a standard slide rule! As with the paper sliding rule, you need to learn how to read a standard slide rule before you can use it.

The slide rule is read much as the paper sliding rule you have been using, but there are some special things to learn. Look at your standard slide rule. You see that it has more than two scales, but we are going to use only the C and D scales. Notice that the graduation marks at the left of these scales are labeled **1**, not 0 as on our sliding rule. These marks are called the **left indexes**.

You recall that when you marked your sliding rule with log values of numbers, the distances between these marks were unequal. Now look at the C and D scales of your slide rule. Notice that they have three major sections in terms of how many graduations there are between the numerals printed in larger type. The first section is from the left index to large 2, the second is from 2 to 4, the third is from 4 to the right end (or right index).

Now think of each of the divisions between the numbers in larger type (1 to 2, 2 to 3, 3 to 4, etc.) as having a value of one whole. Then see how many spaces there are between these numbers. Between 1 and 2 there are 100 small spaces. Each small space has a value of $\frac{1}{100}$ or .01 of the whole. For ease of counting, each tenth space is numbered and has a longer line. Each fifth line is also longer. In Figure O the locations of some numbers are labeled as they would appear on the C scale of a slide rule. Study this figure to find the values for the positions shown by the lettered arrows. Then write these values as asked in SET XXIII on the following page.

Figure O



Remember that by multiplying or dividing by 10 or by powers of 10 all numerals you will need to use can be given values between 1 and 10.

SET XXIII

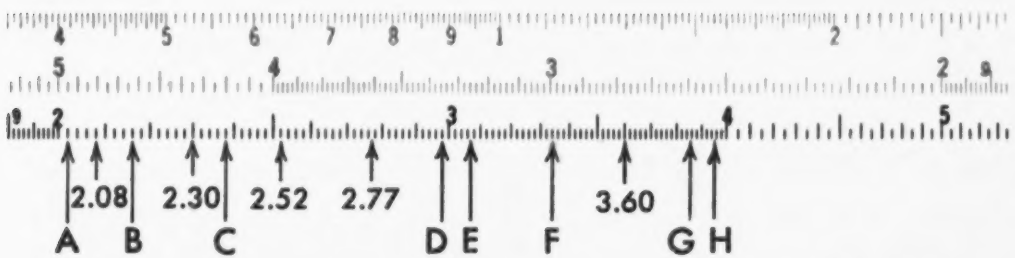
- A is read as _____
- B is read as _____
- C is read as _____
- D is read as _____
- E is read as _____
- F is read as _____

Check your answers on page 70.

The second major section of the scales is between 2 and 4. The distances between 2 and 3 and between 3 and 4 are progressively shorter than the distance between 1 and 2. Also, there are only 50 spaces between numerals. Each of these spaces has a value of $\frac{1}{50}$ or .02 of the whole. Again, each fifth line is longer to make reading easier.

Figure P shows some numbers located on this section of the C scale. Study the figure until you can find the values for the positions shown by the lettered arrows. Then write the values as asked in SET XXIV.

Figure P



Notice that 2.77 is halfway between 2.76 and 2.78. Other values may be read (or estimated) in the same way.

SET XXIV

A is read as _____

B is read as _____

C is read as _____

D is read as _____

E is read as _____

F is read as _____

G is read as _____

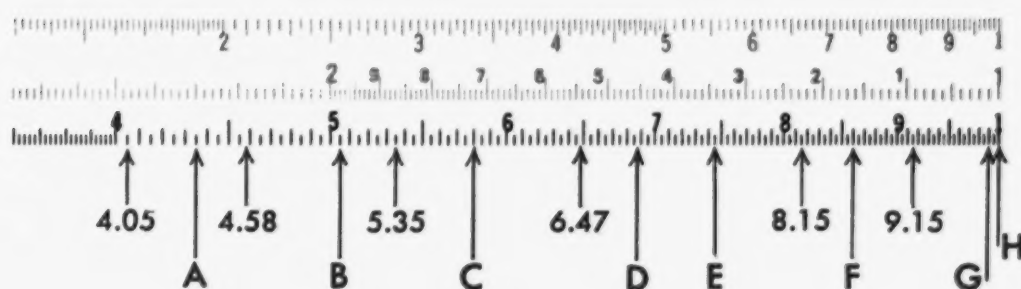
H is read as _____

Check your answers on page 70.

The last section of the scales is from 4 to the right end, or right index. Between numerals the distances continue to get progressively shorter, and now only 20 spaces are marked. Each of these spaces is therefore equal to $\frac{1}{20}$ of one whole, or .05.

Figure Q has some locations marked for this section of the C scale. Study the figure; then do SET XXV.

Figure Q



SET XXV

Write the values of the lettered arrows as shown in Figure Q.

A is read as _____

B is read as _____

C is read as _____

D is read as _____

E is read as _____

F is read as _____

G is read as _____

H is read as _____

Check your answers on page 71.

You have estimated the value of locations between graduation marks before. But notice that in some parts of the slide rule scales (toward the right end of each major section—near 2, near 4, and from 9 on) the marks are closer together. It is somewhat more difficult to estimate locations in these parts of the scales.

Multiplying and Dividing on the Slide Rule

Multiplying and dividing on the standard slide rule are done in much the same way that adding and subtracting were done on the paper sliding rule. We have already had some practice using the simple slide rule we made by marking log values on the sliding rule. Let's try these same steps on the standard slide rule to get better acquainted with it. We will start with the easy example, 2×3 .

First slide the left index (marked 1) of the C scale directly above the first number in our example, 2, on the D scale. You can use the hairline in the "runner" to locate positions if you wish. Second, place the hairline over the second number, 3, on the C scale. Third, under the hairline read the product 6 on the D scale. Figure R shows this placement of the rule.

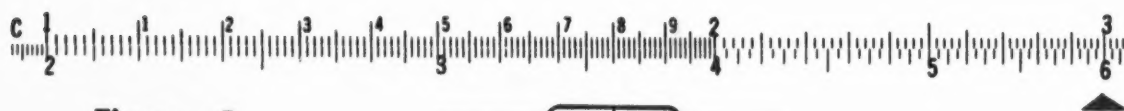
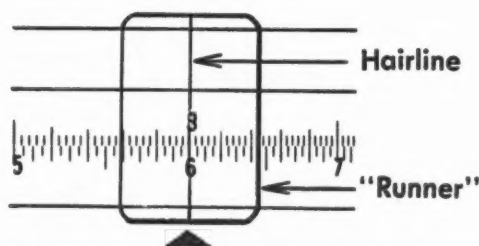


Figure R



Simple, isn't it? Now return the slide to normal position; then try 2×4 . Leave the slide where it is (for finding 2×4) and find the product of 2×2 . Do you get the idea?

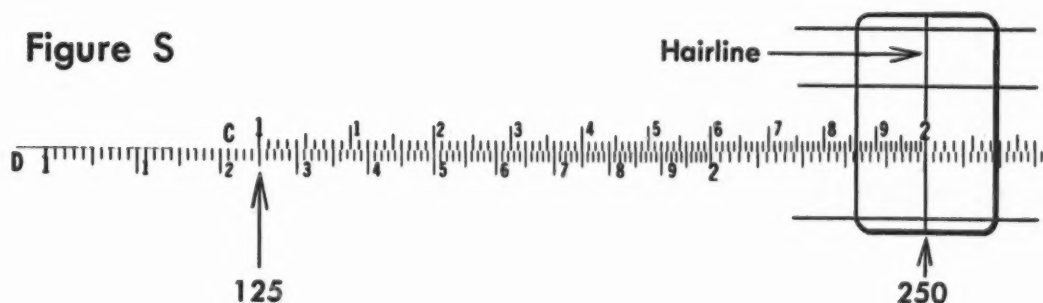
Figure S shows you how to do 125×2 , which you set and read in a somewhat different way. First, where is 125 on the D scale? Move the slide (which has the C scale on it) about halfway to the right and ignore it. Look at the first major section of the D scale, between the larger 1 and 2. The smaller numbers in between stand for 1.1 (or 11, or 110), 1.2, 1.3, . . . 1.9, 2.0. To set 125, place the hairline over the longer line between the small 2 and small 3. How many spaces are there between this setting and the small 2? You see, there are 5, which gives us the 5 of 125.

Now place the left index of the C scale under the hairline. Move the hairline over the large 2 on the C scale. Under the

hairline, read the product on the D scale. Can you read 25? You must remember to estimate to get 250 as the answer. You can estimate the location of the decimal point by using good judgment and see that 250 is the answer.

When you can easily judge how many places there will be in your answer, you do not need to first express your factors in scientific notation to arrive at an estimate. However, in most examples you will do on the slide rule, the complete estimation process is helpful in locating the factors on the rule and in placing the decimal point in the product.

You will need to decide which way you are going to estimate when you do the examples in SET XXVI.



SET XXVI

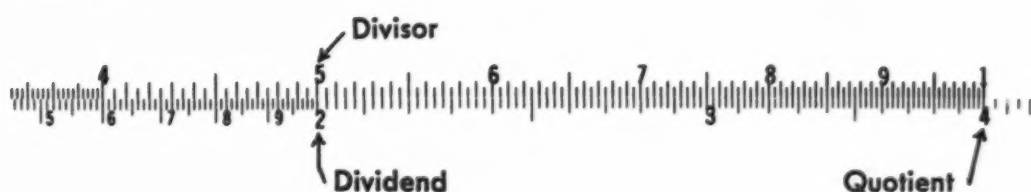
- | a | b | c |
|-------------------------|------------------------|-----------------------|
| 1. $2.7 \times 3.6 =$ | $.019 \times 45 =$ | $.0031 \times 29 =$ |
| 2. $1.71 \times 3.4 =$ | $.214 \times 362 =$ | $41.8 \times .0133 =$ |
| 3. $2.76 \times 4.82 =$ | $6.85 \times 39.4 =$ | $424 \times 9150 =$ |
| 4. $53.5 \times 49.8 =$ | $705 \times 382,000 =$ | $804 \times 304 =$ |
| 5. $81.7 \times 7.96 =$ | $.0239 \times 487 =$ | $9.05 \times 1393 =$ |

Check your answers on page 71.

Using what you've learned about multiplying on the slide rule, now try dividing. Let's look at our easy example again: $2 \times 3 = 6$. The inverse of this is $6 \div 3 = 2$. Set the hairline over the dividend, 6, on the D scale. Slide the divisor, 3, on the C scale under the hairline. Read the quotient, 2, on the D scale under the left index of the C scale. Do you remember using this placement of the rule before? See Figure R.

Sometimes you will need to read your quotient under the right index of the C scale. In Figure T we show the placement of the rule for such an example: $20 \div 5 = 4$. In a simple example like this we do not need to use the complete estimation process since we can easily see where to place the decimal point in the quotient. However, we do need to remember that on the slide rule 2 and 20 are read in the same location.

Figure T



Try the following simple examples to practice reading quotients under the right index of the C scale.

- | | |
|------------------|------------------|
| a. $36 \div 9 =$ | b. $40 \div 8 =$ |
| c. $24 \div 6 =$ | d. $48 \div 8 =$ |
| e. $72 \div 9 =$ | |

You may be ready to do a more difficult example, but let's take it by simple steps: $386 \div 49.8 = ?$

Step 1 Express the example in scientific notation:

$$(3.86 \times 10^2) \div (4.98 \times 10^1) = ?$$

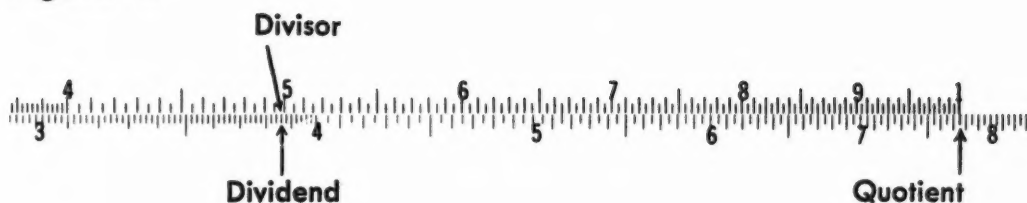
Step 2 Locate the dividend (3.86) on the D scale using the hairline.

Step 3 Locate the divisor (4.98) on the C scale and place it under the hairline.

Step 4 Read the quotient on the D scale below the **right** index of the C scale.

Step 5 Place the decimal point using estimation. Figure U shows placement of the rule for this example.

Figure U



Step 6 Read the quotient from the rule: 7.75. After placing the decimal point by estimation, the final quotient is approximately 7.75.

Study the steps above very carefully. You may want to repeat the same example a number of times before you start a new example. When you feel ready, do SET XXVII. Do all of the examples using the slide rule. You may want to check your work by dividing in the regular way with pencil and paper.

SET XXVII

a. $87.5 \div 37.7 =$

b. $3.75 \div .0227 =$

c. $.685 \div 8.93 =$

d. $1.29 \div 9.70 =$

e. $.00377 \div 5.29 =$

f. $2875 \div 37.1 =$

g. $871 \div .468 =$

h. $.0385 \div .01462 =$

i. $3.14 \div 2.72 =$

j. $3.42 \div 81.7 =$

Check your answers on page 71. If you had difficulty, you may want to find some additional work in your book or in a book of a higher grade level to give you further practice.

You should really feel proud of yourself! You have made great strides in mathematics. Your understanding of the slide rule should be very useful as you move ahead in science and mathematics.

Now that you have developed some skill with the slide rule, use it every chance you get. Use it for checking your work in school. Take it on trips to figure gas mileage. You can even use it to figure batting averages. Occasionally you might want to select word problems from books to work just to keep up your skill. We predict the slide rule will become an indispensable tool for you. You won't be without one!

CHECKING YOUR THINKING

SET I

Likenesses:

1. Each scale is a uniform scale; that is, the intervals between the marks are equal on each scale.
2. The points are numbered in counting order. The values of the numbers increase by one as you move toward the right and decrease by one as you move toward the left.
3. Each line begins and ends with the symbol of three dots (. . .). This symbol stands for "and so on." It is used to indicate a line that extends as far as one can imagine in both directions. Such are the lines which mathematicians think about. A line segment is the part of a straight line that lies between two points on the line. Figures C, D and E are representations of line segments.

Differences:

1. The lengths of the lines are different.
2. Even though all the scales are uniform, the intervals on any one of the scales are different from the intervals on the other scales.
3. The numeral that shows where the scale starts is not the same for each scale. (We can begin a line segment at any point on a line. See examples of this on the opposite page in the answers to SET II.)

SET II



The difference between Mary's age and her brother's is 4 years. Her brother is 4 years older than she; Mary is 4 years younger than he.

You can start the scale for this example with any numeral up to 9. Do you see why?



$4 + 7 = ?$ $4 + 7 = 11$. Jack and Ray walked 11 miles all together.



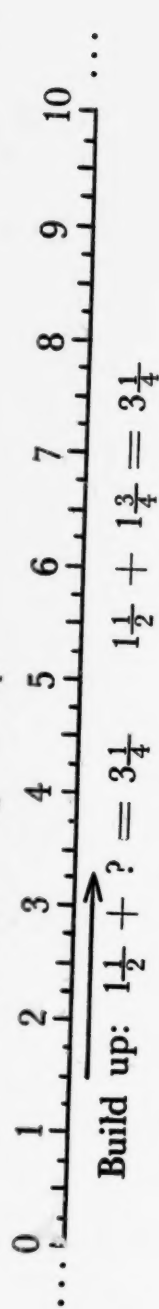
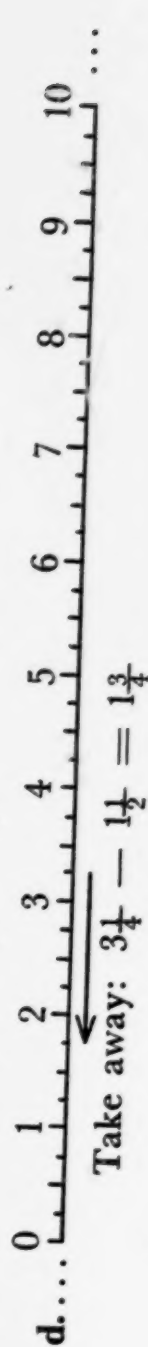
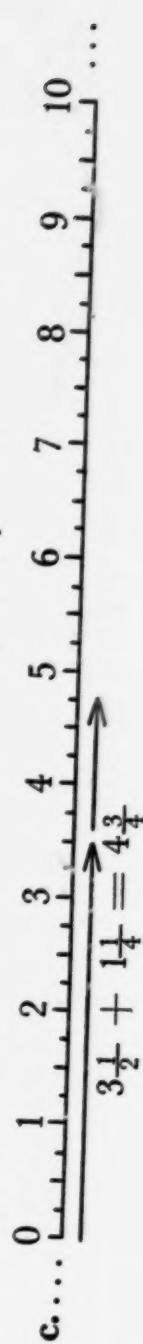
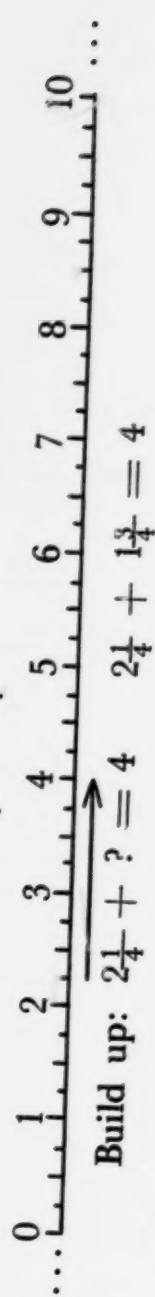
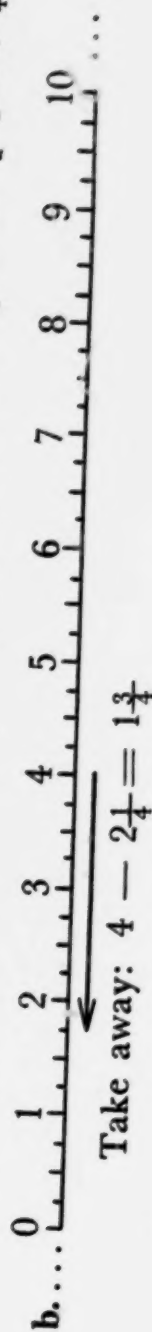
$18 + ? = 21$; $18 + 3 = 21$.

Eric's rope is 3 feet longer than Bob's.

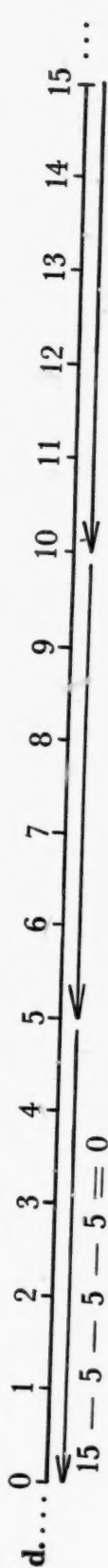
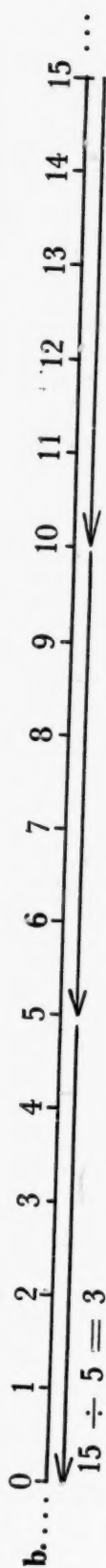
You could start this scale with 18 or any numeral up to 18, and you would not need to extend the scale beyond 21. Think about this.



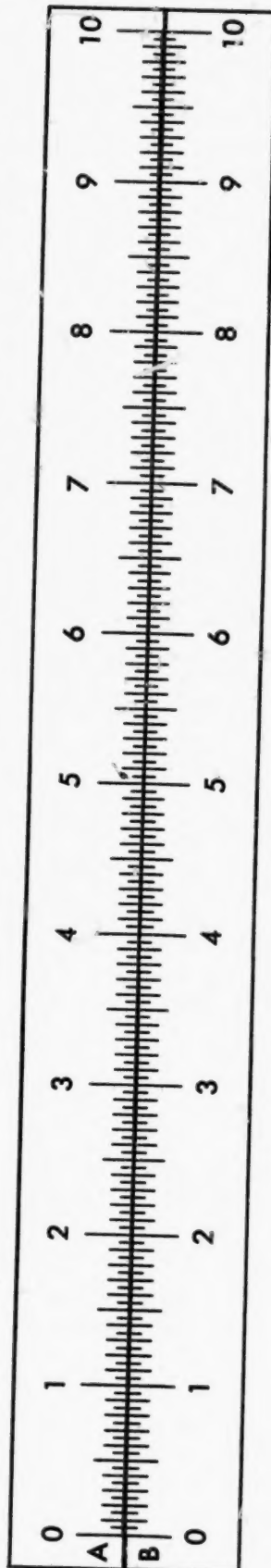
In locating fourths, did you think: $\frac{1}{4}$ is $\frac{1}{2}$ of $\frac{1}{2}$ so it will be $\frac{1}{2}$ of the $\frac{1}{2}$ space; $\frac{3}{4}$ is $\frac{1}{2} + \frac{1}{4}$.



SET IV



Do you see how Examples a and c are related? Do you see how Examples b and d are related?

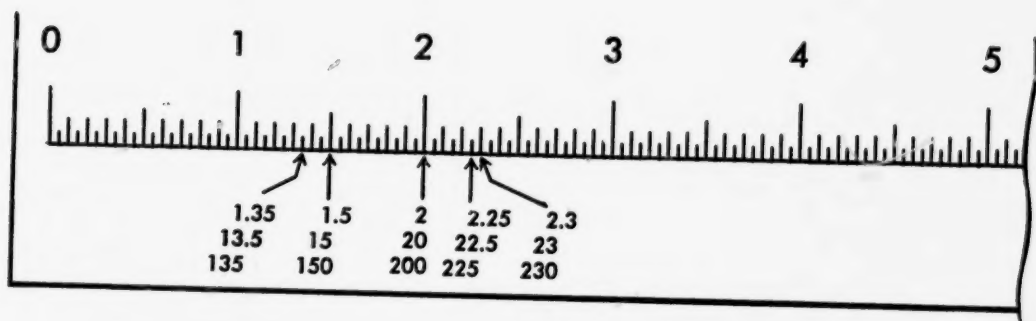


SET V

Check each index and then refer to the labeled diagram below for the correct location in each example.

- a. No index change; 2
- b. Index of 10; $20 \div 10 = 2.0$; or $2.0 \times 10 = 20$; $2.0 = 2$
- c. Index of 100; $200 \div 100 = 2.00$; or $2.00 \times 100 = 200$; $2.00 = 2.0 = 2$
- d. No index change; 1.5
- e. Index of 10; $15 \div 10 = 1.5$; or $1.5 \times 10 = 15$; 1.5
- f. Index of 100; $150 \div 100 = 1.50$; or $1.50 \times 100 = 150$; $1.50 = 1.5$
- g. No index change; 1.35
- h. Index of 100; $135 \div 100 = 1.35$; $1.35 \times 100 = 135$; 1.35
- i. Index of 10; $13.5 \div 10 = 1.35$; $1.35 \times 10 = 13.5$; 1.35
- j. Index of 10; $23 \div 10 = 2.3$; $2.3 \times 10 = 23$; 2.3
- k. No index change; 2.3
- l. Index of 100; $230 \div 100 = 2.30$; $2.30 \times 100 = 230$; $2.30 = 2.3$
- m. No index change; 2.25
- n. Index of 100; $225 \div 100 = 2.25$; $2.25 \times 100 = 225$; 2.25
- o. Index of 10; $22.5 \div 10 = 2.25$; $2.25 \times 10 = 22.5$; 2.25

Can you use an index to locate any position on your sliding rule? If you can, continue.

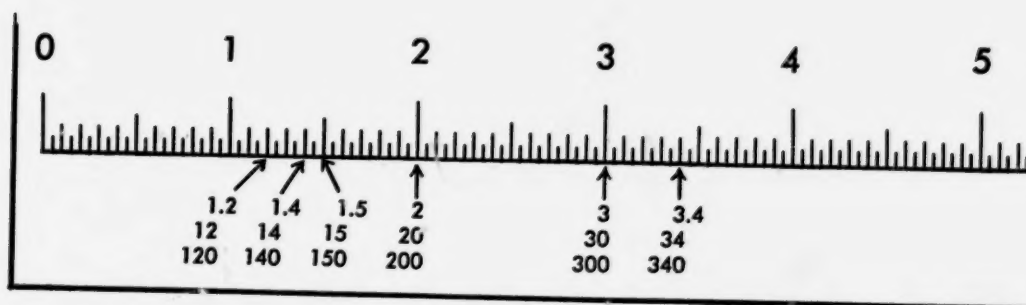


SET VI A

Check index and sums (boldface numerals) and then refer to labeled diagram below to check locations.

- a. No index change; $2 + 3 = \mathbf{5}$
- b. Index of 10; $20 \div 10 = 2.0 = 2$; $2.0 \times 10 = 20$;
 $30 \div 10 = 3.0 = 3$; $3.0 \times 10 = 30$; $20 + 30 = \mathbf{50}$
- c. Index of 100; $200 \div 100 = 2.00 = 2$; $2.00 \times 100 = 200$;
 $300 \div 100 = 3.00 = 3$; $3.00 \times 100 = 300$;
 $200 + 300 = \mathbf{500}$
- d. Index of 10; $12 \div 10 = 1.2$; $1.2 \times 10 = 12$;
 $14 \div 10 = 1.4$; $1.4 \times 10 = 14$; $12 + 14 = \mathbf{26}$
- e. Index of 100; $120 \div 100 = 1.20 = 1.2$; $1.20 \times 100 = 120$;
 $140 \div 100 = 1.40 = 1.4$; $1.40 \times 100 = 140$;
 $120 + 140 = \mathbf{260}$
- f. Index of 10; $15 \div 10 = 1.5$; $1.5 \times 10 = 15$;
 $34 \div 10 = 3.4$; $3.4 \times 10 = 34$; $15 + 34 = \mathbf{49}$

If you are sure of yourself with SET VI A, do SET VI B.

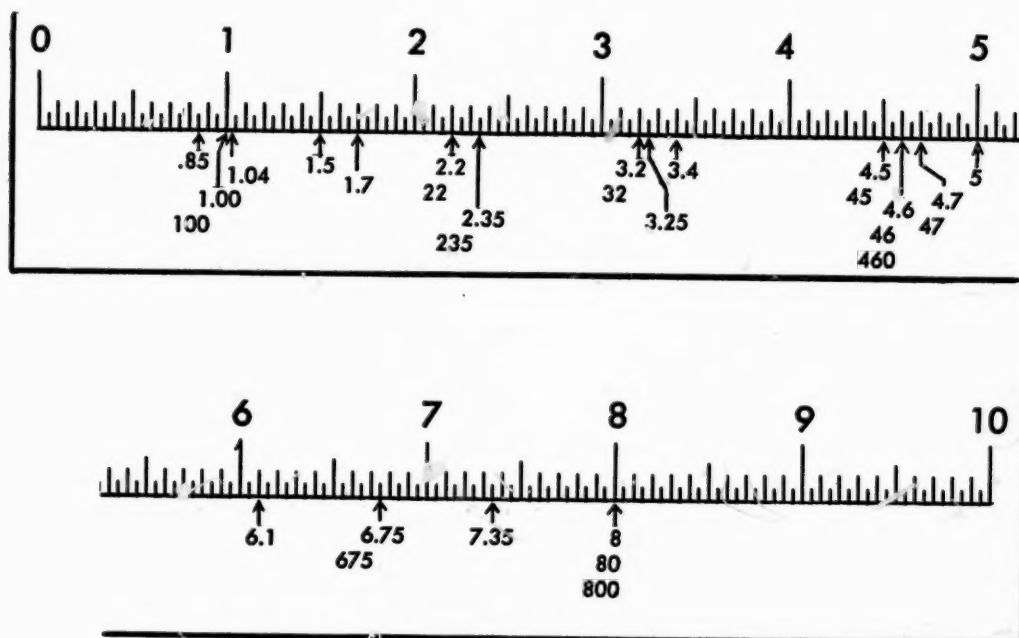


SET VI B

Check index and sums (boldface numerals) and then refer to labeled diagram below to check locations.

- No index change; $1.5 + 3.4 = \mathbf{4.9}$
- Index of 10; $45 \div 10 = 4.5$; $22 \div 10 = 2.2$; $45 + 22 = \mathbf{67}$
- Index of 10; $32 \div 10 = 3.2$; $47 \div 10 = 4.7$; $32 + 47 = \mathbf{79}$
- No index change; $6.1 + 1.7 = \mathbf{7.8}$
- Index of 100; $235 \div 100 = 2.35$; $460 \div 100 = 4.60 = 4.6$;
 $235 + 460 = \mathbf{695}$
- No index change; $1.04 + 7.35 = \mathbf{8.39}$
- No index change; $5.00 = 5.0 = 5$; $3.25 + 5.00 = \mathbf{8.25}$
- No index change; $3.20 = 3.2$; $6.75 + 3.20 = \mathbf{9.95}$
- Index of 100; $800 \div 100 = 8.00 = 8$; $100 \div 100 = 1.00 = 1$; $800 + 100 = \mathbf{900}$
- Since we have to use an index of 100 for 675 to change it to 6.75, we must use the same index of 100 for 85; $85 \div 100 = .85$, which is located between 0 and 1 on the scale.
 $85 + 675 = \mathbf{760}$

If you had trouble with SET VI B, reread pages 14 through 18 and do SET VI A and then SET VI B again. If you had no trouble with SET VI B, continue.



SET VII

Did you think as follows?

- a. $7 + 5$ is more than 9 so the example involves carrying; therefore I will use the right (10) end of the A scale. I place the right end of the A scale over the 7 of the B scale. I see that the 3 of the A scale is extending over the end of the B scale, $7 + 3 = 10$. I find the other addend (5) on the A scale. I read the number directly below the 7 on the B scale which is 2. I add the 10 to the 2 to get the sum 12.
 $7 + 3 + 2 = 7 + 5 = \mathbf{12}$
- b. $4 + 8$ will involve carrying so I use the right end of the A scale. $4 + 6 + 2 = 8 + 4 = \mathbf{12}$ If you had difficulty with this example, review the steps on page 20.
- c. $3 + 6$ does not involve carrying so I use the left (0) end of the A scale. $3 + 6 = \mathbf{9}$

Do you understand how to add examples with carrying on your sliding rule? If so, continue.

SET VIII

Did you think as follows?

- a. Index of 10; $39 \div 10 = 3.9$; $76 \div 10 = 7.6$;
 $3.9 + 7.6 = 11.5$; $11.5 \times 10 = 115$
- b. No index change; $4.7 + 3.9 = 8.6$
- c. Index of 10; $2.8 + 9.3 = 12.1$; $12.1 \times 10 = 121$
- d. Index of 10; $6.7 + 5.9 = 12.6$; $12.6 \times 10 = 126$
- e. Can locate addends as less than 1, or multiply each addend by 10; $.68 \times 10 = 6.8$; $.93 \times 10 = 9.3$;
 $6.8 + 9.3 = 16.1$; $16.1 \div 10 = 1.61$
- f. Index of 100; $8.50 + 4.70 = 13.20$; $13.20 \times 100 = 1320$
- g. Index of 100; $3.80 + 9.50 = 13.30$; $13.30 \times 100 = 1330$
- h. Index of 100; $2.55 + 9.45 = 12.00$; $12.00 \times 100 = 1200$
- i. Index of 100; $7.05 + 8.50 = 15.55$; $15.55 \times 100 = 1555$
- j. No index change; $9.05 + 8.85 = 17.90$

If you had difficulty locating the numbers on your sliding rule, reread pages 14 through 17 before continuing.

SET IX

- a. Index of 10; $8.7 - 4.5 = 4.2$; $4.2 \times 10 = 42$
- b. No index change; $9.3 - 2.8 = 6.5$
- c. Index of 100; $1.78 - .86 = .92$; $.92 \times 100 = 92$
- d. Index of 10; $5.9 - 3.4 = 2.5$; $2.5 \times 10 = 25$
- e. No index change; $6.1 - 2.9 = 3.2$
- f. Index of 100; $7.05 - 6.35 = .70$; $.70 \times 100 = 70$
- g. Index of 100; $6.35 - 2.85 = 3.50$; $3.50 \times 100 = 350$
- h. Index of 100; $1.45 - .97 = .48$; $.48 \times 100 = 48$
- i. No index change; $3.85 - 2.65 = 1.20$
- j. No index change; $1.90 - .85 = 1.05$

Table of Some Powers of 10

$$10^1 = 10$$

$$10^2 = 10 \times 10 = 100$$

$$10^3 = 10 \times 10 \times 10 = 1000$$

$$10^4 = 10 \times 10 \times 10 \times 10 = 10,000$$

$$10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100,000$$

$$10^9 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1,000,000,000$$

SET X

Yes, in each case the numbers expressed in scientific notation were correct. Did you think as follows?

- a. 967,000 has to be divided by 10 five times (10 is used as a factor 5 times) or by 100,000 to get an integer that is 1 or more but less than 10.
- b. 338,000,000 has to be divided by 10 eight times, or by 100,000,000.
- c. 345 has to be divided by 10 two times, or by 100.
- d. 238.13232 has to be divided by 10 two times, or by 100.

SET XI

- a. $1,000,000,000 = 1 \times 10^9$
- b. $93,000 = 9.3 \times 10^4$
- c. $265,000 = 2.65 \times 10^5$
- d. $3,890,000 = 3.89 \times 10^6$
- e. $6,000,000,000,000 = 6 \times 10^{12}$
- f. $6.9 \times 10^5 = 690,000$
- g. $3 \times 10^7 = 30,000,000$
- h. $6.35 \times 10^3 = 6350$
- i. $5.463 \times 10^4 = 53,630$
- j. $6.3 \times 10^2 = 630$

Check by multiplying:

- a. $1 \times 10^9 = 1 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$ or
 $1 \times 1,000,000,000$
- b. $9.3 \times 10^4 = 9.3 \times 10 \times 10 \times 10 \times 10$ or
 $9.3 \times 10,000$
- c. $2.65 \times 10^5 = 2.65 \times 10 \times 10 \times 10 \times 10 \times 10$ or
 $2.65 \times 100,000$
- d. $3.89 \times 10^6 = 3.89 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$ or
 $3.89 \times 1,000,000$
- e. $6 \times 10^{12} = 6 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$
 $10 \times 10 \times 10$ or $6 \times 1,000,000,000,000$

SET XII

Did you think as follows?

- a. To change .0647 ($\frac{647}{10,000}$) so that the numerator has a single digit to the left of the decimal point I need to divide by 100; $\frac{647}{10,000} = \frac{6.47}{100} = \frac{6.47}{10 \times 10} = \frac{6.47}{10^2}$
- b. $.000398 = \frac{398}{1,000,000}$; divide both terms by 100;
 $\frac{398}{1,000,000} = \frac{3.98}{10,000} = \frac{3.98}{10 \times 10 \times 10 \times 10} = \frac{3.98}{10^4}$
- c. $.247 = \frac{247}{1000}$; divide both terms by 100; $\frac{247}{1000} = \frac{2.47}{10^1}$
- d. $.000005 = \frac{5}{1,000,000} = \frac{5}{10 \times 10 \times 10 \times 10 \times 10 \times 10} = \frac{5}{10^6}$
- e. $.00349 = \frac{349}{100,000}$; divide both terms by 100;
 $\frac{3.49}{1000} = \frac{3.49}{10 \times 10 \times 10} = \frac{3.49}{10^3}$
- f. $35,000,000 = 3.5 \times 10^7$
- g. $964,000 = 9.64 \times 10^5$
- h. $.00539 = \frac{539}{100,000}$; divide both terms by 100;
 $\frac{5.39}{1000} = \frac{5.39}{10 \times 10 \times 10} = \frac{5.39}{10^3}$
- i. $94.347 = 9.4347 \times 10^1$
- j. $903.45 = 9.0345 \times 10^2$

SET XII (cont'd.)

k. $\frac{6.7}{10^2} = \frac{6.7}{100} = .067$

l. $\frac{3.943}{10^3} = \frac{3.943}{1000} = .003943$

m. $5.43 \times 10^3 = 5.43 \times 1000 = 5430$

n. $\frac{8.003}{10^2} = \frac{8.003}{100} = .08003$

o. $\frac{3.958}{10^3} = \frac{3.958}{1000} = .003958$

p. $\frac{6.894}{10^5} = \frac{6.894}{100,000} = .00006894$

q. $1.45 \times 10^7 = 1.45 \times 10,000,000 = 14,500,000$

r. $\frac{2.593}{10^3} = \frac{2.593}{1000} = .002593$

s. $\frac{9.993}{10^1} = .9993$

t. $8.395 \times 10^2 = 8.395 \times 100 = 839.5$

SET XIII

a. $10^2 \times 10^3 = 10^5$ b. $10^3 \times 10^5 = 10^8$

c. $10^5 \times 10^7 = 10^{12}$ d. $10^8 \times 10^3 = 10^{11}$

SET XIV

- a. 1. Express multiplier and multiplicand in scientific notation.
 $392 = 3.92 \times 100 = 3.92 \times 10^2$
 $483 = 4.83 \times 100 = 4.83 \times 10^2$
 Example is now $(3.92 \times 10^2) \times (4.83 \times 10^2)$
2. Round 3.92 to 4 and 4.83 to 5. Example is row $(4 \times 10^2) \times (5 \times 10^2)$ or $4 \times 5 \times 10^2 \times 10^2$
3. Complete the multiplication: $4 \times 5 = 20$; $10^2 \times 10^2 = 10^4$;
 $20 \times 10^4 = 200,000$
4. 189,000 is closer to 200,000 than 1,890,000 or 18,900 is, so the answer is approximately 189,000.
- b. 1. $88.3 = 8.83 \times 10^1$; $570,000 = 5.7 \times 10^5$
2. Round 8.83 to 9; 5.7 to 6; $(9 \times 10^1) \times (6 \times 10^5)$ or $9 \times 6 \times 10^1 \times 10^5$
3. $9 \times 6 = 54$; $10^1 \times 10^5 = 10^6$; $54 \times 10^6 = 54,000,000$
4. 50,300,000 is closer to 54,000,000 than is either 503,000,000 or 5,300,000.
- c. 1. $8240 = 8.24 \times 10^3$; 8.39 doesn't change. Why?
2. 8.24 rounds to 8; 8.39 rounds to 8
3. $(8 \times 10^3) \times 8 = 8 \times 8 \times 10^3 = 64 \times 10^3 = 64,000$
4. 69,100 is closer to estimate (64,000) than is 691,000 or 6910.
- d. 1. $45,000 = 4.5 \times 10^4$; 3.14 doesn't change.
2. 4.5 rounds to 5; 3.14 rounds to 3
3. $(5 \times 10^4) \times 3 = 5 \times 3 \times 10^4 = 15 \times 10^4 = 150,000$
4. 141,000 is closer to estimate than is 1,410,000 or 14,100.

SET XV

- a. $10^7 \div 10^2 = \frac{\cancel{10} \times \cancel{10} \times 10 \times 10 \times 10 \times 10 \times 10}{\cancel{10} \times \cancel{10}} = 10^5$ ($7 - 2 = 5$)
- b. $10^9 \div 10^3 = \frac{\cancel{10} \times \cancel{10} \times \cancel{10} \times 10 \times 10 \times 10 \times 10 \times 10 \times 10}{\cancel{10} \times \cancel{10} \times \cancel{10}} = 10^6$
- c. $10^2 \div 10^5 = \frac{\cancel{10} \times \cancel{10}}{\cancel{10} \times \cancel{10} \times 10 \times 10 \times 10} = \frac{1}{10^3}$

Answers shown with the sign $\approx$ are approximate; $\approx$ can be read "is approximately."

SET XVI

- a. 1. $36,700 = 3.67 \times 10^4$; $578 = 5.78 \times 10^2$
 2. 3.67 rounds to 4; 5.78 rounds to 6
 3. $(4 \times 10^4) \times (6 \times 10^2) = (4 \times 6) \times (10^4 \times 10^2) = 24 \times 10^6 = 24,000,000$
 4. 21,200,000 is closest to the estimate (24,000,000).
 5. Approximate product is 21,200,000;
 $36,700 \times 578 \approx 21,200,000$
- b. 1. $685 = 6.85 \times 10^2$; $.342 = 3.42 \times \frac{1}{10^1}$
 2. 6.85 rounds to 7; 3.42 rounds to 3
 3. $(7 \times 10^2) \div (3 \times \frac{1}{10^1}) = \frac{7}{3} \times 10^2 \div \frac{1}{10^1}$
 4. $\frac{7}{3} = 2\frac{1}{3}$ or 2.33; $10^2 \div \frac{1}{10^1} = 10^2 \times \frac{10^1}{1} = 10^3$;
 $2.33 \times 10^3 = 2330$
 5. 2330 is the estimated quotient; $685 \div .342 \approx 2000$
- c. 1. $.359 = 3.59 \times \frac{1}{10^1}$; $48,000 = 4.8 \times 10^4$
 2. 3.59 rounds to 4; 4.8 rounds to 5
 3. $(4 \times \frac{1}{10^1}) \times (5 \times 10^4) = (4 \times 5) \times (\frac{1}{10^1} \times 10^4) = 20 \times 10^3 = 20,000$
 4. 17,200 is closest to 20,000 (the estimated product).
 5. $.359 \times 48,000 \approx 17,200$

SET XVI (cont'd.)

- d. 1. $468 = 4.68 \times 10^2$; $.00583 = 5.83 \times \frac{1}{10^3}$
2. 4.68 rounds to 5; 5.83 rounds to 6
3. $(5 \times 10^2) \times (6 \times \frac{1}{10^3}) = (5 \times 6) \times (10^2 \times \frac{1}{10^3}) = 30 \times \frac{1}{10^1} = 3.0$
4. 2.73 is closest to the estimate 3.0
5. $468 \times .00583 \approx 2.73$
- e. 1. $938 = 9.38 \times 10^2$; $26,500 = 2.65 \times 10^4$
2. 9.38 rounds to 9; 2.65 rounds to 3
3. $(9 \times 10^2) \div (3 \times 10^4) = (\frac{9}{3}) \times (\frac{10^2}{10^4}) = 3 \times \frac{1}{10^2} = \frac{3}{10^2} = .03$
4. .0354 is the approximate quotient since it is closer to the estimate, .03, than is .354 or .00354.
5. $938 \div 26,500 \approx .0354$

SET XVII

- | | |
|--|---|
| a. $.0876 \div .324 \approx .270$ | b. $348 \times .683 \approx 238$ |
| c. $.0056 \times .038 \approx .000213$ | d. $.369 \div .072 \approx 5.13$ |
| e. $56,700 \div 2380 \approx 23.8$ | f. $.695 \times 4590 \approx 3190$ |
| g. $.078 \times 3460 \approx 270$ | h. $9.26 \times .045 \approx .417$ |
| i. $34.500 \times 68.300 \approx 2360$ | j. $.00824 \times .036 \approx .000296$ |

If you had difficulty with any example, find a similar one in SET XVI and review the steps as described in the answers to SET XVI. Then try again to work any that you missed. If you need further explanation, review Part III. When you feel sure of yourself, you are ready to go on with the rest of Part III.

SET XVIII

- | | |
|------------------------|------------------------|
| a. $\log 1.44 = .1584$ | b. $\log 3.03 = .4814$ |
| c. $\log 6.05 = .7818$ | d. $\log 5.49 = .7396$ |
| e. $\log 7.38 = .8681$ | f. $\log 3.60 = .5563$ |
| g. $\log 5.87 = .7686$ | h. $\log 9.40 = .9731$ |
| i. $\log 6.39 = .8055$ | j. $\log 5.43 = .7348$ |

SET XIX

- | | |
|---------|---------|
| a. 1.05 | b. 1.78 |
| c. 4.06 | d. 6.40 |
| e. 9.26 | f. 9.86 |
| g. 5.57 | h. 4.39 |
| i. 7.57 | j. 4.00 |

SET XX

- | | |
|-----------|----------|
| a. 8.49 | b. 11.8 |
| c. 10,500 | d. 776 |
| e. 38.0 | f. 44.1 |
| g. 44,100 | h. 53.7 |
| i. 3000 | j. .0278 |

SET XXI

- | | |
|----------|-----------|
| a. 2.44 | b. 4.60 |
| c. .609 | d. .528 |
| e. 52.8 | f. 8.30 |
| g. .830 | h. .00424 |
| i. .0151 | j. 83,100 |

If you had difficulty with any of the examples in SETS XX or XXI, restudy the steps described in Part III and try the ones you missed again. When you feel sure of yourself, go ahead with Part IV. At last you are ready to begin using a regular slide rule!

SET XXII

- | a | b |
|--------------------|---------|
| 1. 4.5 | 6. 1.5 |
| 2. 7.0 | 7. 3.0 |
| 3. 6.0 | 8. 300 |
| 4. 30.0 (see note) | 9. 300 |
| 5. 90 | 10. .02 |

Note: In Example 4 did you use the **right** end of the rule?

SET XXIII

- | | |
|-------------------|-------------------|
| A is read as 1.08 | B is read as 1.3 |
| C is read as 1.52 | D is read as 1.8 |
| E is read as 1.93 | F is read as 1.99 |

There are 100 spaces ($\frac{1}{100}$ or .01) between large 1 and large 2. Reading from left index 1, you have: 1.00, 1.01, 1.02, 1.03, 1.04, 1.05 (longer line), 1.06, 1.07, 1.08, 1.09, 1.10, 1.11, 1.12, 1.13, . . . 1.19, 1.20, 1.21, 1.22, . . .

SET XXIV

- | | |
|-------------------|-------------------|
| A is read as 2.02 | B is read as 2.16 |
| C is read as 2.38 | D is read as 2.98 |
| E is read as 3.07 | F is read as 3.34 |
| G is read as 3.86 | H is read as 3.95 |

There are 50 spaces ($\frac{2}{100}$ or .02) between 2 and 3 and between 3 and 4, so you read: 2.00, **2.02**, 2.04, 2.06, 2.08, 2.10, 2.12, . . . Or you can estimate halfway locations between marks to get: 2.01, 2.03, 2.05, 2.07, 2.09; . . . 3.01, 3.03, 3.05, **3.07**, 3.09, . . .

SET XXV

- | | |
|-------------------|-------------------|
| A is read as 4.35 | B is read as 5.05 |
| C is read as 5.8 | D is read as 6.87 |
| E is read as 7.45 | F is read as 8.6 |
| G is read as 9.9 | H is read as 10.0 |

There are 20 spaces ($\frac{5}{100}$ or .05) between the numerals on the scales from 4 to the right index 1. You can read: 4.00, 4.05, 4.10, 4.15, . . . 4.95, 5.00, 5.05, . . . 10.00 (right index). In some parts of this section you can estimate locations, as: . . . 4.40, 4.45, 4.50, 4.55, **4.57**, 4.60, . . . 4.95; 5.00, . . . ; 6.00, 6.05, . . . 6.40, 6.45, **6.47**, 6.50, . . . 6.80, 6.85, **6.87**, 6.90, 6.95, 7.00, . . .

SET XXVI

a	b	c
1. 9.72	.855	.09
2. 5.82	77.5	.556
3. 13.3	270	3,880,000
4. 2660	269,000,000	244,000
5. 650	11.6	12,600

SET XXVII

- | | |
|------------|----------|
| a. .232 | b. 165 |
| c. .0767 | d. .133 |
| e. .000713 | f. 77.5 |
| g. 1860 | h. 2.63 |
| i. 1.15 | j. .0419 |

This Independent Learning Booklet is part of an experimental project to provide mathematics materials for able pupils and to develop a mathematics program adapted to individual differences. The project is based on several studies and exploratory programs carried on in San Diego County during the past several years, such as those sponsored by the San Diego Metropolitan Education Study Council and the Assistant Superintendents Group. Objectives of this phase of the project are to stimulate and widen pupil interest in various areas of mathematics; to develop deeper insights into mathematical principles, concepts, and relationships; to introduce aspects of modern mathematical thinking; and to develop motivation for able pupils to continue to study and explore mathematics and related fields.

The project is coordinated by the Department of Education, San Diego County, and directed by a steering committee selected from members of the San Diego Metropolitan Education Study Council. Support for the project is derived from funds provided under Title III of the National Defense Education Act of 1958 and matching funds from the participating school districts and the San Diego County Department of Education.

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